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COMMERCIAL
SERIES

ELEMENTS OF BUSINESS ARITHMETIC

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MACMILLAN'S COMMERCIAL SERIES

ELEMENTS OF BUSINESS ARITHMETIC

BY

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PREFACE

THE preparation of the text which follows was undertaken in the belief that the arithmetic of the schools should teach the methods most in vogue in the business world, and that those methods should be so taught as to form correct habits in those who are to attack the problems of real life.

The accomplishment of these ends has involved, first, an investigation into the methods of the various fields of business activity, and, second, the writing of the whole subject from the point of view of habit-forming rather than from that of either the conventional or the scientific treatment.

The methods chosen are believed to have the sanction of usage by those in the business world best qualified to speak. The manner of presentation has been tested by nearly ten years of use, in manuscript form, in the schools and classes of which the authors have had charge.

The processes presented are strictly arithmetical. No form of domination by higher studies is more insidious or harmful than the attempt to apply the abstractions of algebra and geometry to the problems of arithmetic for the use of children. Immature young people do not comprehend these abstractions and can only memorize them and apply them haltingly. On the other hand, if they fully understand the concrete methods of arithmetic and can understandingly solve its problems, their minds are better equipped with those concrete concepts which alone give meaning to the more abstract forms and truths of the higher branches of mathematics.

While the methods used in this book are those of the counting room, the shop, or the farm, the pure mathematical

element has not been sacrificed. When the mathematical reasoning of available methods is clear, the chief considerations have been, just as they are in the business world, shortness of operation, quickness of solution, and the minimum of opportunity or likelihood of error. These considerations have not always led to formal methods, equally applicable to all contingencies, but have rather pointed to direct ways of solving the kind of problems most frequently met. While the emphasis has not been, therefore, upon the best methods for infrequent and unusual problems, their solution is none the less clearly prepared for.

Topics admittedly obsolete have been omitted, while others less used than formerly have received correspondingly less emphasis. In general, only those topics or phases of topics have been treated which are applicable to present-day problems, and in the order of their need, regardless of the traditional arrangement and sequence of subject matter.

To those who believe that there should be a re-canvassing of the whole field of arithmetic at the close of the grammar school, or in the first high school year, with more attention to the methods used in actual life and with the deliberate purpose of developing an habitual mode of attack which seeks the most direct and accurate methods of solution, we commend our book in the hope that it may be of real service. We are firmly convinced that there is a large and growing number of school men and women who believe that this is the only road to adequate and practical results from arithmetic teaching.

We desire to express appreciation of the sympathetic criticism and coöperation of the editor of the series in which the book appears, and of the many courtesies shown us by officers and employees of numerous large business houses whom we consulted in our quest for information as to current arithmetic practices.

A. H. B.
W. A. A.

EDITOR'S INTRODUCTION

Two small boys were overheard in conversation as they went out from the morning assembly exercises of a given school, just after the thirteenth chapter of First Corinthians had been read. One's remark to the other fairly expressed the estimate which has been placed on the value of the elementary school studies. The lad's comment was, "Readin, ritin, and rithmetic, and the greatest of these is rithmetic." Arithmetic has claimed a large part of the school's time; it has presented the very citadel of difficulty to the pupil; and it has been made the object of first importance by the teacher.

While the above statements are true there are other facts in striking contrast. Results at present secured from the study of arithmetic are most unsatisfactory. Several investigations in different parts of the country have shown that pupils go out from the schools not understanding the processes of modern business, and not able to make trustworthy computations. Schools are not realizing an educational result from arithmetic commensurate with the time and effort spent on it, and the present situation is one which calls for careful consideration.

In the first place arithmetic is of all the elementary school studies the one most in danger of becoming conventionalized. Teacher and textbook tend to perpetuate themselves. Marked changes may occur in practical affairs about which the teacher and the textbook author know little. Thus it was that the mercantile methods of the eighteenth cen-

tury continued as the basis of arithmetical instruction in the nineteenth century, long after those methods had disappeared; and thus there are traces of the eighteenth century still to be discerned in the arithmetics of the present. If the merchant of an earlier generation were to reappear and attempt to do business, he would be dumfounded by the changes in commercial procedure; but he could scarcely be at a greater disadvantage than are pupils who are trained after the methods which he had used and then sent out to take their places in the world of to-day.

The book herewith presented is an honest attempt to set forth correctly the fundamental operations of modern business, and to furnish a goodly amount of drill on the kind of computations which make up present-day commercial practice. Messrs. Bigelow and Arnold have spared no pains to inform themselves on current business operations, and the editor believes that they have presented their information succinctly and logically arranged for purposes of instruction. The book has been the result of much labor in its first preparation, and as first prepared it was duplicated to serve as a text, and modified in class instruction for several years. In addition to this it has been revised and adapted in accordance with suggestions of experienced teachers in different parts of the country. It is believed that all this has resulted in a book of accurate information, of sound mathematical basis, and of high teaching quality.

Several features of the book will commend themselves to teachers. Among these are the script illustrations as models for trial balances, ledger accounts, time sheets, accounts of sales, etc. These have been executed by the skilful pen artist and illustrator, Mr. E. C. Mills.

Chapter IV on Fractional Parts presents a natural and easy approach to fractions, and will be found of great practical value. The authors have had the courage to put deci-

mals ahead of common fractions, treating them as they should be treated, simply as a descending scale in our decimal notation, of which whole numbers are an ascending scale.

The book will be found to have little of the impractical and troublesome G. C. D. and L. C. M. problems. Fractions of large denominations are not introduced, as they present difficulties and are almost never encountered except in the arithmetics. Square root and the treatment of mensuration are disposed of in connection with weights and measures. The antiquated percentage problems to be solved by the use of formulas are omitted, as are the conventional percentage formulas themselves. Partial payments is relegated to an unimportant place in the Chapter on Interest. The so-called true discount is eliminated from the textbook as it is from business; and partnership and proportion are so treated as to bring them within the practice of the actual world.

In some particulars the book is not as revolutionary as the authors would have desired, but it is believed to be as revolutionary as it could be without breaking with the practice of the schools. This text, it is believed, will prove a logical and easy completion of the average elementary course in arithmetic. It should find a place in the last years of the grammar school, as the finishing book of the ungraded school, and for the first high school year.

Throughout, this book will be found to use the method of concrete presentation. The pupil is constantly asked to consider problems with the thought of determining the particular solution which will get a result the most directly. Thus there is an absence of any "wooden" working by rule. At every turn the pupil is required to select his solution, and to use his head in applying the form selected. This cannot fail to produce clear thinking and a facility in computations which will give accuracy. The problems of easy solution for mental arithmetic offer one valuable feature

of the book. These, largely used, will develop power of accurate thought and power of expression.

A limited range of treatment, with freedom of explanation and plenty of drill on the fundamentals; not too much arithmetic attempted, but what is attempted done well; not generalized and abstract number, but arithmetic related to the life experiences of the child; problems selected from the world around about the child; not a treatment which shall be "milk for babes," but one which will afford such a ruggedness of drill as will make arithmetic a means of disciplinary education and a book of accurate information, — these are the standards which the "Elements of Business Arithmetic" has sought to meet. Possibly it has failed in some particular, but it is published in the confident belief that there is a large place for such a presentation as is here attempted. The authors and the editor have the satisfaction of having worked long and faithfully. They invite corrections and suggestions for the improvement of the book.

C. A. H.

CONTENTS

THE ELEMENTS OF BUSINESS ARITHMETIC

I

ADDITION AND SUBTRACTION

1. Combinations to 9. Pupils who use this book will probably know the combinations of numbers up to 9, but it may be presumed that they are not sufficiently quick and accurate in the use of these combinations. Thorough drills should be given and continued until pupils can think groups as a whole and can instantly name sums without naming the parts.

NOTE. — There are but forty-five possible combinations of numbers up to 9. The common device of having each one of the combinations on a separate card and giving results at sight rapidly and in varied order, forms a practical drill. Each card may have the same combination on its back with the order of the figures reversed.

DRILL TABLE

The forty-five two-figure combinations. Name sums at sight.

7	4	2	4	1	3	4	3	3	1	4	2	2	1	1	8	9	8	5	6	4	5	5
7	6	5	3	7	6	2	3	2	5	1	2	1	3	1	9	9	8	5	1	4	3	4

7	1	5	6	6	8	9	8	7	7	4	9	7	6	7	5	3	2	4	5	7	6	
2	8	6	6	9	6	1	2	3	5	8	3	8	7	9	9	8	9	9	8	4	2	

2. Combinations to 19. The following combinations should be drilled upon and learned in the same way as those in Sec. 1. They are important for rapid addition (Sec. 3).

The instant calling of results should be insisted upon. Drill cards as above suggested will be useful.

NOTE. — In making these combinations, the second number should be thought of as one ten and a given number of units and not as a given number of units and one ten. Thus, in combining 14 and 13 we should think of 24 and 3; in combining 17 and 18 of 27 and 8.

DRILL TABLE

Forty-five more combinations, 11 to 19. Name sums at sight.

12	13	11	17	11	17	12	15	12	16	19	17	11	14	15	12	19
15	18	11	17	16	14	12	16	13	13	14	13	14	15	17	11	16
15	18	17	12	16	15	18	17	13	16	11	19	14	19	16	19	13
15	18	16	17	12	13	16	18	11	16	19	18	13	15	14	19	13
14	14	18	18	19	12	18	17	17	12	11						
14	18	12	11	13	14	15	11	19	19	15						

3. Adding by Groups. In adding columns of figures one should think first of the sum of each group, then group the sums and name the sum of the groups. In the problem given, think and name the sum of the first group as 14 without naming the figures of the group, the sum of the second group as 13, then the sum of 13 and 14 is thought and named. Seeing the next group as 10, we think the sum of 27 and 10, saying 37; then, 11, 48. In adding, name the results only, as: 14, 27, 37, 48.

8	48
3	
7	37
3	
8	27
5	
9	14
5	
48	

NOTE. — Double drill may be had from the same problems by adding both up and down; *e.g.* 11, 21, 34, 48. This is also a way of detecting errors. If the same result is obtained from adding both ways, the sum is probably correct.

4. Adding Two Columns. By practice, two columns of figures may be readily added at once. The tens should be combined first and the units added to their sum (Sec. 3).

The problems below may be used for practice in both single and double column adding. Add both up and down. Drill until the group sums can be named quickly.

Add.

37	95	67	68	56	45	27	35	46	24	99	48	71	89	76	32	33	45
40	36	76	66	75	72	88	87	72	98	76	79	88	15	90	54	38	98
63	36	76	97	57	83	33	77	32	68	71	90	13	31	16	58	84	17
84	66	54	78	86	96	38	55	33	19	10	34	42	34	86	92	10	57
67	88	38	86	84	48	47	94	65	56	25	56	75	85	83	17	42	56
22	71	95	84	32	23	47	85	65	76	81	34	56	87	34	50	60	70
71	95	84	32	23	47	85	65	76	81	8	78	19	30	54	76	58	43
45	24	98	76	67	19	14	67	32	78	76	14	98	83	28	34	13	65

Practice adding by groups.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
180	718	678	432	178	852	458	37	458	123	7689	23	1768
717	678	743	327	187	918	1079	1876	7	7453	4	717	324
324	187	509	834	778	415	2345	324	6789	1064	318	6324	5324
873	548	791	130	543	915	718	98	4317	8796	1745	6	1098
376	876	987	734	435	478	3458	1756	5692	187	678	1	367
324	578	713	105	791	698	1234	304	2934	67	5487	75	84
943	528	281	375	905	768	5432	32	701	5432	34	150	69
473	432	958	104	548	785	7981	7615	45	478	91	235	75
432	958	104	548	785	442	2701	89	6194	91	3605	768	150
573	473	109	423	478	871	71	432	5061	2398	148	23	1785

Add across. Add down. Problems like these are useful in training to accuracy, and frequently occur in offices. If the final results are not alike, the student should find and correct mistakes without awaiting correction by the the teacher.

1. $324 + 625 + 463 + 526 = ?$
$462 + 375 + 836 + 472 = ?$
$378 + 643 + 728 + 645 = ?$
$643 + 372 + 654 + 528 = ?$
$584 + 653 + 936 + 364 = ?$
$? + ? + ? + ? = ?$

2. $103 + 184 + 216 + 135 + 320 = ?$
$413 + 204 + 311 + 401 + 405 = ?$
$764 + 835 + 735 + 631 + 987 = ?$
$367 + 543 + 263 + 413 + 187 = ?$
$185 + 371 + 213 + 715 + 478 = ?$
$? + ? + ? + ? + ? = ?$

3. $265 + 167 + 324 + 734 + 178 = ?$
$562 + 713 + 432 + 817 + 384 = ?$
$276 + 810 + 305 + 978 + 308 = ?$
$417 + 187 + 523 + 734 + 198 = ?$
$672 + 432 + 278 + 598 + 471 = ?$
$? + ? + ? + ? + ? + ? = ?$

5. Adding Long Columns. *Civil Service Method.* In adding long columns of figures it is often desirable to retain the exact sum of each column. Errors are more easily located and unnecessary re-adding of columns which are correct is often avoided. Generally speaking, this practice should always be used for columns of more than ten numbers.

$791.52
504.83
879.26
243.79
732.46
47.95
856.43
497.65
541.26
616.72
857.94
51
6.3
53
51
60
$6569.81

This is sometimes known as the "Civil Service Method," presumably from its large use in government offices. It may be frequently used to advantage in many offices. Practice using this method in the problems given after Sec. 6.

NOTE.—*Drill in writing*, from dictation, long columns of numbers of varied size, so that neat vertical columns will be secured. Verify answers by adding downward, if the first addition was upward, or *vice versa*.

6. Addition Proof by Check. In counting-rooms or elsewhere where long columns of figures are common, some system for checking the correctness of results of addition is often resorted to. Such devices are of various kinds, but all consist of some variation of a casting out process, as of the 9's or 11's.

The *Unitate* method is here presented. It consists of removing all the nines from the numbers added and from the sum. The remainder would necessarily be less than nine and being a single figure is termed a *unitate*. *When the unitate of the numbers added is the same as the unitate of the sum, the addition is usually correct.*

The process of finding the unitate of a given number or, as it is often called, "casting out the nines," is based on the facts that our system of notation is a decimal one and that there is but a single unit of difference between 9 and the basis of our notation, 10.

Every digit in our notation stands for the number of tens (or powers of ten) represented by the digit. Now there are as many nines in each of these as there are tens and there would be a remainder of a single unit from each ten. But as each digit represents the number of tens, there would be a number of units equal to the digit itself remaining, after all the nines had been taken out. The sum of the digits of any number would represent, then, the number of units remaining after all the nines had been "cast out." If this sum is a number with more than one digit, it still contains a nine and the sum of its digits would be the true remainder. When this remainder consists of but one digit, it is the desired unitate.

To find the unitate of 87,564,892, add the digits and their sum is 49; adding these gives 13, and adding these gives 4, the unitate.

Thus,

	Unitates.
32,543	8
16,789	3
11,942	2
27,683	1
38,578	5
96,541	3
73,847	5
297,923	5

In theory, the unitates of every addend are themselves reduced to a unitate, which should equal the unitate of the correct answer.

In practice, the unitate of the first addend (8) is written and then added to the digits of the second addend and the whole reduced to its unitate (3). This is again added to the digits of the third addend and the third unitate is found (2). This process is continued until the unitate of the last addend is found, which should be the same as the unitate of the answer.

Checking by the unitate is not an absolute proof. It will not detect an error in transposition of digits, or where the wrong digits total the same as the right ones. It is a satisfactory check, however, in most cases.

PROBLEMS

Solve and prove the following:

1.

Trial Balance, June 1, 19—.

		Debit	Credit
1	A. E. Hill, Proprietor		3042 70
2	Merchandise	2416 20	
3	Notes Receivable	1145 50	
3	Cash	669 50	
3	Furniture and Fixtures	377 00	
4	Horses and Wagon	465 00	
5	Shipment #1, J. A. Bently	370 00	
5	Shipment #1, J. E. Amos	72 00	
6	Expense	95 00	
6	Interest and Discount	70 00	
17	G. A. Frank	267 00	
17	Dunn & Company	225 00	
8	Notes Payable		845 00
19	Consignment #1, Wm. Frances		302 00
19	W. E. Dewey		95 00
19	O. M. Bates		300 00
20	Merchants Supply Co.		337 50
20	A. C. Fairwell		779 00
20	Anaconda Construction Co.		466 00

2.

BAUSCH & LOMB OPTICAL COMPANY
MANUFACTURERS

Rochester, N. Y. Dec 12, 19—

Sold to J. R Melton
Unison, Mo

1 lb. Granulated Zinc		25
5 kg. Soft Glass Tubing Size 8 mm	3	00
1 kg. " " " " 10 mm		60
24 ft. Rubber Tubing Size 8 mm	2	70
12 ft. " " " 10 mm	2	70
12 Safety Tubes 300 mm	1	35
18 Test Tube Clamps	2	70
1 Hoffmans Apparatus Decom. Water	3	00
1 gm. Platinum Wire		75
6 Porcelain Crucibles	1	13
1 Burette	1	13
1 blk. Metric Weights	1	60
10 Reagent Bottles 125 cc. KCl	1	12
10 " " 125 cc. K_2SO_4	1	13
10 " " 125 cc. KNO_3	1	12
10 " " 125 cc. NH_4OH	1	13
10 " " 125 cc. NaOH	1	12
10 " " 125 cc. KOH	1	13
12 Chlorides of Calcium Tubes	1	35
1 doz. Rubber Stoppers		30

3.

GRAND RAPIDS, MICH., April 23, 19—

THE HANCOCK FURNITURE COMPANY
356-359 STATE STREET

SOLD TO Board of Education
Toledo, Ohio

TERMS Cash

1	Roll Top Desk		60	00
1	Index File		35	00
12	Office Chairs	1.50	18	00
1	#2 Copying Press		12	00
1	Office Safe		175	00

Received payment May 1, 9—
The Hancock Furniture Co.
By E. C. Mills

4. List of appropriations by Congress, for biennium.

	19__	19__
Deficiencies	$ 19,651,968.25	$ 25,083,395.78
Legislative, Executive, and Judicial	27,598,653.66	28,558,258.22
Sundry Civil	61,763,709.11	49,968,011.34
Support of the Army	77,888,752.83	77,070,300.88
Naval Service	81,876,791.43	97,505,140.94
Indian Service	8,540,406.77	9,447,961.40
Rivers and Harbors	20,228,150.99	10,872,200.00
Forts and Fortifications	7,188,416.22	7,518,192.00
Military Academy	652,748.67	973,947.26
Pensions	139,847,600.00	138,360,700.00
Consular and Diplomatic	1,968,250.69	2,020,100.69
Agricultural Department	5,978,160.00	5,902,040.00
District of Columbia	8,638,097,00	11,018,540.00
Miscellaneous	3,025,064.95	2,860,828.52
Total		

5. The cotton crop of the United States by states for five years.

States	1	2	3	4	5
North Carolina	480,000	400,000	426,000	504,000	400,000
South Carolina	960,000	874,000	948,000	955,000	845,000
Georgia	1,448,000	1,226,000	1,493,000	1,498,000	1,405,000
Florida	54,000	57,000	56,000	60,000	55,000
Alabama	1,161,000	1,136,000	1,287,000	1,065,000	1,040,000
Mississippi	1,776,000	1,349,000	1,460,000	1,418,000	1,385,000
Louisiana	577,000	651,000	851,000	864,000	832,000
Texas	3,143,000	2,575,000	2,682,000	2,575,000	2,446,000
Arkansas	921,000	665,000	771,000	938,000	855,000
Tennessee	381,000	240,000	229,000	303,000	255,000
All others	334,000	267,000	498,000	578,000	516,000
Total crop					

6. Add down. Add across.

7654	4753	1954	1763	8197	6548	?
1458	3674	1756	3263	3954	3245	?
5786	9876	1753	5132	6587	3642	?
4327	9876	587	3674	5743	6798	?
56	7815	2301	1567	4326	189	?
1587	9817	3246	1685	8542	7432	?
1563	6743	9816	1076	875	1904	?
8543	3425	178	25	3607	98	?
5674	8795	5432	5768	5843	2345	?
? +	? +	? +	? +	? +	? =	?

7. Departmental sales for the week ending June 15, 19—.

Days	Clothing	Dry Goods	Furnishings	Millinery	Groceries	Total
Monday	695.50	894.30	175.65	325.45	678.10	?
Tuesday	546.15	716.98	243.25	817.42	313.48	?
Wednesday	981.76	654.32	145.60	567.89	543.26	?
Thursday	578.90	765.10	324.65	687.58	987.60	?
Friday	842.45	918.75	216.40	561.46	674.15	?
Saturday	985.50	818.40	456.12	764.55	925.48	?
Total	?	?	?	?	?	?

8. The records of a post-office show the following mail for six days: Monday, registered letters, 625; ordinary letters, 14,570; postal cards, 2134; book packets, 957; parcels, 184; newspapers, 25,514. Tuesday, registered letters, 541; ordinary letters, 13,576; postal cards, 2134; book packets, 587; parcels, 146; newspapers, 26,156. Wednesday, registered letters, 750; ordinary letters, 14,569; postal cards, 3456; book packets, 1056; parcels, 178; newspapers, 24,356. Thursday, registered letters, 587; ordinary letters, 13,452; postal cards, 2451; parcels, 143; newspapers, 23,781. Friday, registered letters, 547; ordinary letters, 13,567; postal cards, 1346; book packets, 890; parcels, 157; newspapers, 26,543.

Saturday, registered letters, 857; ordinary letters, 15,472; postal cards, 3145; book packets, 789; parcels, 245; newspapers, 23,100.

Arrange these facts in tabular form, in six columns, under proper headings. Find the total number of pieces of mail for each day, the total number of pieces of each class, and the total number for six days.

7. Method of Subtraction. Whether the pupil subtracts by the method of borrowing from the minuend or by adding to the subtrahend, is immaterial. Ordinarily he should be permitted to use the method previously taught him. The effort should be toward facility, which may only be acquired by practice. The examples here given are suggestive rather than sufficient. If when these are completed there is not the power of quick and accurate subtraction, much additional work should be given.

If another method is desired, that known as the addition method is suggested. It has the advantage of using the addition combinations previously taught and involves no new process.

$$\begin{array}{r} 2069 \\ -\ 984 \\ \hline 1085 \end{array}$$

In the accompanying problem, we say 4 and 5 are 9, writing the 5. Then, 8 and 8 are 16, writing the 8 and carrying the 1 as in addition. Then, 10 (9 + 1) and 0 are 10, writing 0; and 1 (carried from 10) and 1 are 2, writing 1.

8. Making Change. The addition method of making change is the one most generally used by tellers, and is unquestionably the most accurate. Thus, a ten-dollar bill is tendered in payment of a bill of $2.85. Counting out successively a five-cent piece, a dime, two dollars, and a five-dollar bill, the sums are named as follows: $2.85, $2.90, $3, $4, $5, $10.

Name the coins and bills, and the amount of change to be given in each of the following:

1. From $1, for a bill of: 17¢, 43¢, 65¢, 72¢, 28¢, 10¢, 15¢, 40¢, 35¢, 87¢.

2. From $2, for a bill of: $1.26, $1.40, $1.47, $1.61, $1.75, $1.55, $1.69, $1.83, $1.19, $1.33.

3. From $5, for a bill of: $3.50, $1.12, $2.32, $3.37, $2.87, $0.79, $4.11, $1.78, $3.56, $2.75, $4.15.

4. From $10, for a bill of: $3.50, $4.75, $6.32, $7.28, $4.87, $9.15, $8.55, $6.70, $1.95, $4.28.

5. From $20, for a bill of: $17.35, $14.32, $13.45, $10.75, $9.15, $3.85, $12.10, $14.30, $8.24.

9. Horizontal Subtraction. Subtract the following without rearranging. Find the sum of the minuends, the sum of the subtrahends, and the sum of the remainders.

1.

3,264,873	− 286,729	= ?
328,629	− 124,962	= ?
729,687	− 638,469	= ?
382,962	− 146,702	= ?
2,678,212	− 1,476,388	= ?
729,326	− 384,578	= ?
368,742	− 176,386	= ?
504,726	− 386,275	= ?
?	− ?	= ?

2.

496,827	− 268,794	= ?
1,986,702	− 1,346,825	= ?
2,787,543	− 1,968,729	= ?
79,843	− 67,983	= ?
4,869,625	− 2,278,631	= ?
967,198	− 798,631	= ?
472,398	− 126,535	= ?
848,716	− 432,567	= ?
?	− ?	= ?

3.

$867.50	− $742.25	= ?
329.87	− 124.68	= ?
1768.42	− 938.89	= ?
2762.48	− 1262.34	− ?
3786.32	− 439.37	= ?
9623.29	− 3674.28	= ?
9627.42	− 7672.91	= ?
1076.16	− 729.78	= ?
?	− ?	= ?

4.

$6487.50	− $3228.25	= ?
329.75	− 128.95	= ?
6756.50	− 2278.36	= ?
4798.60	− 3128.75	= ?
12986.72	− 9647.22	= ?
3678.45	− 2968.42	= ?
728.32	− 648.25	= ?
2198.65	− 1297.70	= ?
?	− ?	= ?

Find the new balances in the following banking individual ledger accounts by adding the deposits to the balances and subtracting the checks. Find the total of balances, checks, and deposits. Prove.

1.

NAMES	BALANCES	CHECKS	DEPOSITS	BALANCES
Ames, Wm. E.	865.52	321.56	675.80	——— —
Bentley, C. A.	584.32	127.55	220.00	——— —
Cromer, G. L.	954.60	532.58	276.50	——— —
Dayton, F. R.	523.40	305.45	560.00	——— —
Elsworth, Geo.	976.35	532.78	125.40	——— —
Frank, G. A.	126.65	38.72	423.80	——— —
Gramm, D. C.	925.43	413.86	575.94	——— —
Hughes, C. M.	1214.34	615.47	213.44	——— —
Innes, U. P.	752.30	456.87	435.87	——— —
Justin, John	178.95	34.56	698.00	——— —
King, A. S.	986.57	485.74	325.34	——— —
	?	?	?	?

2.

NAMES	BALANCES	CHECKS IN DETAIL	TOTAL CHECKS	DEPOSITS	BALANCES
		125.50			
Anson, E. M.	5671.80	232.20	——— —	546.70	——— —
Barnes, T. D.	1544.42	678.90	——— —	546.70	——— —
		750.00			
Cowles, E. M.	2345.60	142.76	——— —	756.05	——— —
		525.00			
Doyle, F. E.	5467.80	453.50	——— —	854.32	——— —
Farish Bros.	967.85	134.75	——— —	768.90	——— —
Grim, G. L.	1267.98	1100.00	——— —	987.45	——— —
		190.00			
Haines, F. R.	845.34	253.78	——— —	1200.00	——— —
		1234.56			
Johns, B. I.	3289.07	26.78	——— —	584.32	——— —
Love, P. V.	876.35	325.40		542.08	
	?	?	?	?	?

10. Problems for Explanation. In problems where the processes themselves are very simple, the more difficult work of accurate expression in equation form and of explanation should be carefully taught. Exactness in language is the only proper expression for an exact science.

A suggestive statement and explanation are given. Set forms of explanation are not desirable, but a correct use of mathematical symbols and of clear and accurate English in the explanation of a problem should be insisted upon. Use the fewest words possible to express the thought clearly.

PROBLEM. — During one season a jobbing carpenter built five dwellings which cost him respectively $3176, $5194, $1342, $6950, and $788. He received for building them $3875, $6820, $1280, $7896, and $875. What were his season's profits?

$$\$3176 + \$5194 + \$1342 + \$6950 + \$788 = \$17{,}450.$$
$$\$3875 + \$6820 + \$1280 + \$7896 + \$875 = \$20{,}746.$$
$$\$20{,}746 - \$17{,}450 = \$3296, \text{ season's profits.}$$

EXPLANATION. — The total cost of building the five buildings is the sum of $3176, $5194, $1342, $6950, and $788, or $17,450. The total amount received is the sum of $3875, $6820, $1280, $7896, and $875, or $20,746. His profits, therefore, are the difference between $20,746 and $17,450, or $3296.

PROBLEMS

1. A grain dealer bought 15,640 bushels of wheat, and sold at one time 3465 bushels, at another time 4205 bushels, and at another time 1080 bushels. How many bushels remained?

2. A man deposited in a bank $9672. He drew out at one time $4234, at another $1700, at another $762, and at another $49. How much remained?

3. A teacher's salary was $1200. His living expenses were $760. He paid $314 for a lot and $95 for a horse. How much of his salary remained?

4. A merchant in a year bought goods to the amount of $8750. He paid for clerk hire $6735, and for rent $318. For how much must he sell his goods in order to clear $1250?

5. I sold a farm for $9625 and a house for $3275. Lost $475 on the farm and gained $360 on the house. What did each cost me? What was the total gain or loss?

6. The population of Indiana in 1900 was 2,516,462. The population of the principal cities of the state was as follows: Indianapolis, 169,164; Evansville, 59,007; Fort Wayne, 45,115; Terre Haute, 36,673; South Bend, 35,999. How much did the population of the state exceed that of these cities?

7. The distance from Chicago to Buffalo is 523 miles, and from Chicago to New York 980 miles. How far is Buffalo from New York?

8. At a sawmill 120,000 feet of pine lumber were sawed in a month. 47,250 feet of it were sold to one man and 32,575 to another. How much of the month's output remained?

9. The imports of sugar and molasses into the U.S. in one year amounted to $108,387,388, and ten years later to $101,100,000. What was the amount of decrease?

10. Three persons bought a hotel valued at $45,675. The first agreed to pay $8575, the second twice as much as the first, and the third the remainder. How much was the third to pay?

11. Borrowed of a bank at one time $875, at another $385, and at another $528. Having paid $1275, how much do I owe?

12. A minister had his life insured for $5000. At the time of his death, $375 of his salary was unpaid; he owned a farm worth $4675, but upon it was a mortgage of $2385, and his small debts amounted to $879. What was the value of his estate?

13. A stock dealer bought 789 cattle from A, and 1249 from B. He then sold 228 to C, 468 to D, and the remainder to E. How many did E buy?

14. A merchant commenced business with $7500. The first year he gained $1275, the second year he lost $2475, the third year he gained $978, and the fourth year lost $674. How much had he left at the end of the fourth year?

15. A bank had $422,785 on hand. During the day they received on deposit $14,657, and paid out by check $24,570. How much remained on hand at the close of the day?

16. The cost of my house and lot was $12,860. I expended $1367 for carpenter work, $567 for bricklaying, $6850 for plumbing, $587 for painting, and $369 for sodding and fencing the grounds. I then sold the property at a loss of $135, receiving $7850 in cash and a note for the balance. What was the face of the note?

17. During five years a firm gained $36,750. The first year they gained $7565; the second $4125; the third as much as both the first and second years; and the fourth year the difference between the gains of the first and second years. How much did they gain the fifth year?

18. The exports of cattle from the United States during a period of nine years were as follows: $159,179; $439,987; $1,103,095; $13,344,195; $14,304,103; $12,906,693; $31,161,131; $30,445,249; and $39,099,095. During a later period of seven years; $23,032,428; $33,461,022; $30,603,796; $34,560,672; $36,357,451; $37,827,500; and $30,516,833. For which period were the exports greater, and how much?

19. The expenditures for schools during one year in Alabama were $1,583,250; in Arizona, $377,253; in Arkansas, $1,396,594; in California, $6,401,439; and in Illinois, $18,167,219. How much more was expended in Illinois than in the other states mentioned?

20. Find the balance of the following:—

First National Bank, Dayton, O. In account with *Wm. Spitler*

Jan.	2	Deposit	594	50	Jan.	3		25	00
	7	"	225	00		4		85	00
	8	"	654	00		4		30	00
	10	Check 22	543	25		5		67	00
	12	" 27	188	00		7		95	00
	13	Deposit	1129	50		14		40	25
	17	"	174	34		15		90	06
	18	"	53	75		17		118	25
	22	"	547	50		20		37	500
	23	"	67	50		23		17	50
	24	"	355	75		25		13	50
	27	Check 25	87	54		28		35	75
	28	" 28	786	50		29		1486	68
	29	Deposit	34	75		30		543	20
	30	"	1453	25		30	Balance (Red Ink)		

II

MULTIPLICATION AND DIVISION

11. Reference and Drill Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	152	168	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

12. Suggestions for Study. It is presumed that students pursuing this course already know the multiplication tables through the 12's. It will prove of great practical value to know them *well* through the 20's. If undertaken, the pupil should build his own tables and *learn* them. Drill on them

should not stop short of absolute mastery. A half-known table of 15's will seldom be used.

NOTE. — Drill cards of convenient size, with the factors on one side and the product on the other, are valuable aids in securing readiness and accuracy. They are equally usable in multiplication and factoring (or division), by varying the sides shown.

In learning a table, it should be kept in mind that the product is the same regardless of the order of the factors. Thus, 136 is the product of both 8×17 and 17×8. If this is kept in mind, the new combinations to be learned in the higher tables become constantly less; *e.g.* if all the tables through the 19's are known, the only new combination in the 20's to be learned is, $20 \times 20 = 400$.

After completing the 12's to 20, it will be found helpful to pursue the following order:

Review the 5's and 10's and study the 15's and 20's.
Review the 3's, 6's, 9's, 12's, and take the 18's.
Review the 4's, 8's, 12's, and take the 16's.
Then take the 13's, 14's, 17's, and 19's.

Problems for practice in short multiplication and division with abstract numbers may be drawn from the above table or dictated to students at will.

13. Suggestions and Problems for Explanation.

PROBLEM. — A horse worth \$130 and 3 cows worth \$36 each, were exchanged for sheep at \$6 per head and \$82 in money. How many sheep were received?

$\$36 \times 3 = \$108.$
$\$108 + \$130 = \$238$, value of horse and cows.
$\$238 - \$82 = \$156.$
$\$156 \div \$6 = 26$, the number of sheep received.

EXPLANATION. — If 1 cow was worth \$36, 3 cows were worth three times \$36, or \$108. If the cows were worth

$108 and the horse $130, together, they would be worth the sum, or $238. If $82 was received in money, the difference between $238 and $82, or $156, was received in sheep. If each sheep was worth $6, as many would be received as $6 is contained in $156, or 26. (See Sec. 10.)

Two kinds of division problems. Division, as commonly used, includes two classes of problems, *e.g.*:

(1) When it is required to find how many *times* one quantity is contained in another, and

(2) When it is desired to find the *size* of one of the equal parts of the quantity. As examples, the following may be given:

1. How many bags will be required to hold 42 bushels of wheat, if each bag will hold 3 bushels?

2. How many bushels in each bag, if 42 bushels are put into 14 bags?

In (1) the dividend and divisor are the same in kind.

In (2) the dividend and the result are the same in kind.

For the purposes of clear thinking and rigid explanation, it is thought best to treat problems like (2) as problems in fractions.

To assist in keeping the reasoning clear, it is suggested that problems like (1) be written in equation form in this way:

42 bu. ÷ 3 bu. = 14, the number of bags required.

Problems like (2) should be written:

$\frac{1}{14}$ of 42 bu. = 3 bu., number in each bag.

It should be noted, however, that the results in both classes of problems are obtained by the same process.

In the written solution of a problem, great care should be taken to use the equation correctly.

An equation should be true in kind as well as in amount.

Concrete numbers should invariably be named, and should be written first in the equation.

The sign of multiplication is the St. Andrew's Cross. It should be read "multiplied by." If the word *times* is used, the second number is read first. Thus, the first equation above should read, \$ 36 multiplied by 3, or 3 times \$ 36.

READING. — When the reasons for the steps of the problem are well known by the pupil, much time may be gained by a mere reading of the equations instead of requiring an explanation.

The above statement would be read as follows: \$ 36 multiplied by 3 is \$108; \$108 plus \$130 is \$238; \$ 238 less \$ 82 is \$ 156; and \$ 156 divided by \$ 6 is 26, or the number of sheep received.

14. Multiplication by Numbers Greater than 20. Multiplication, when the multiplier is greater than one's known multiplication table, involves the use of partial products or the product of each figure of the multiplier and the whole of the multiplicand. The essential thing to be observed in the process is that the right-hand figure of each partial product should be directly under each multiplier. The reason for this is easily seen when we remember that the units figure is multiplied first in each case. Thus, units times units give units, and units multiplied by hundreds are hundreds, etc., the result each time being of the same denomination as that of the figure multiplied by. The fact that these partial products are to be added to find the complete product explains the necessity of arranging units under units and tens under tens.

5236 bu.
932
10472
15708
47124
4879952 bu.

When the multiplier contains a cipher, it is passed over and the next figure to the left becomes the multiplier. Care should be taken to begin the partial product under the new multiplier.

When ciphers are to the right of either the multiplier or multiplicand, or both, the last figures to the right of both (other than the ciphers) are placed under each other. The ciphers are disregarded in the partial products, but as many are placed to the right of the product as are to the right of both numbers being multiplied.

15. Long Division. If the quotient is placed above the dividend, the reason for the place of each figure will be more readily seen. Thus, there are 9 *hundred* thirty-fours in 325 hundreds, and accordingly the 9 is placed directly above the hundreds of the dividend; likewise 34 into 197 tens gives *tens*, and should be placed directly over the tens of the dividend, etc.

$$\begin{array}{r} 958 \\ 34\overline{)32572} \\ 306 \\ \hline 197 \\ 170 \\ \hline 272 \\ 272 \\ \hline \end{array}$$

The ease with which one gets to know the denomination of any partial quotient will often prevent getting absurd answers, and will greatly simplify the "pointing off" process in decimals to be met later.

Problems in which the divisor contains ciphers to the right may be much shortened by cutting off as many figures to the right of the dividend as there are ciphers to the right of the divisor. The figures so cut off are always the whole or part of the remainder, and if written fractionally should be written over the entire divisor.

$$\begin{array}{r} 19\frac{25}{1500} \\ 15\cancel{00}\overline{)285\cancel{25}.} \end{array}$$

The process of long division may be much shortened by subtracting the partial products as we find them, writing only the remainder each time. Drill on this will not only be found valuable as a mental training, but of real value in lessening work.

$$\begin{array}{r} 1463 \\ 52\overline{)76088} \\ 240 \\ 328 \\ 168 \\ \end{array}$$

12, remainder.

PROBLEMS

1. 7668 ÷ 36.	**11.** 1,674,918 ÷ 1980.	**21.** 372,104 ÷ 386.
2. 48,967 ÷ 52.	**12.** 324,217 ÷ 268.	**22.** 385,200 ÷ 1370.
3. 20,982 ÷ 78.	**13.** 356,686 ÷ 682.	**23.** 466,830 ÷ 2730.
4. 24,476 ÷ 58.	**14.** 1,769,824 ÷ 3800.	**24.** 8,326,900 ÷ 1029.
5. 10,908 ÷ 236.	**15.** 367,240 ÷ 461.	**25.** 9,230,021 ÷ 3120.
6. 98,340 ÷ 63.	**16.** 378,625 ÷ 325.	**26.** 780,164 ÷ 119.
7. 76,055 ÷ 53.	**17.** 148,050 ÷ 315.	**27.** 27,552 ÷ 328.
8. 42,400 ÷ 98.	**18.** 986,172 ÷ 186.	**28.** 172,096 ÷ 3600.
9. 39,628 ÷ 76.	**19.** 34,572 ÷ 129.	**29.** 34,216 ÷ 9203.
10. 97,266 ÷ 78.	**20.** 3,467,000 ÷ 360.	**30.** 1,647,756 ÷ 198.

Solve, expressing in equation form, and explain:

1. Bought 568 bushels of corn at 48¢ a bushel and 675 bushels of wheat at 76¢ a bushel. What did both cost?

NOTE. — While 48¢ is the real multiplicand, the problem is shortened by treating it (being the smaller number) as the multiplier. In expressing the work in an equation, however, 48¢ must appear as the multiplicand.

2. Mr. McFarland has property valued at $6700. He buys land and sells it at a gain of $5 per acre. He is then rated at $8075. How many acres did he buy?

3. A stock buyer having $3540 buys 16 horses at $75 each, and invests the remainder of his money in cattle at $36 per head. How many head of cattle does he buy?

4. At the rate of 45 miles an hour, how long will it take a train to run 325 miles?

5. If a book contains 255 pages, and there are 1864 ems to the page, how many ems in the book?

6. An electric railway company has 435 miles of track which was built at a cost of $83,672 per mile. What was the total cost of constructing the road?

7. A coal dealer bought 428 tons of coal at $7.50 per ton. He sold 200 tons of it at $8 a ton and the remainder at $8.75. Find his gain.

8. A man sold mining stock for $9600, a mill for $12,600, and three houses for $12,530, $6780, and $9870 respectively. He invested $21,975

of the proceeds in the stock of a manufacturing company at $75 a share, and the balance in railroad stock at $80 a share. How many shares of each did he buy?

9. If a clerk receives $1850 a year, and pays $38 a month for board and room, $2.40 a month for laundry, $3.80 a month for life insurance, $12 a month to a building and loan association, $376 a year for clothing, and $325 a year for incidentals, how much does he have to invest in business after a term of ten years?

10. Into how many states as large as Texas (265,780 sq. mi.) could the United States (3,616,484 sq. mi.) be divided?

11. A manufacturer bought 165 tons of steel billets at $32 a ton, 45,000 pounds of steel bars at $1.80 a hundred, and 75 tons pig iron at $19 a ton. How much did he pay for all?

12. A young man takes out a life insurance policy for $2000 and agrees to pay $3.78 a month for 20 years. How much money will he have paid to the company at the end of that time?

13. How many cords of 128 cu. ft. in a pile of wood containing 122,880 cu. ft.? What is it worth at $3.50 per cord?

14. A dealer bought 370 tons of coal by the long ton (2240 lb.) at $5 a ton. He retailed it at $7 a short ton (2000 lb.). What was his total gain? What would he have gained if he had sold it at the same price per long ton?

15. A stock dealer sold 26 carloads of cattle, 26 head of cattle in each car, at 3¢ a lb. If the cattle averaged 935 lb., how much did he receive for them?

16. I paid $12,400 for apples at $2.50 per barrel. The total loss in storage was 38 barrels, and I paid 5¢ a barrel for storage and 2¢ a barrel for drayage. How much did I gain, if I sold them for $3.80 per barrel?

17. A certain oil company produced 23,000,000 barrels of refined oil in one year. How many tanks each holding 35,000 barrels would it take to store this oil? On one of the company's farms there was stored in tanks of the above capacity 2,450,000 barrels of crude oil. How many tanks were there?

18. If the area of all the continental divisions of the earth is 51,238,800 sq. mi. and the population is 1,487,900,000, how many people are there to the sq. mi.?

III

DECIMALS

16. Decimal Notation. By *decimal*, is meant a *tenth*. Our notation (Arabic) is said to be a decimal notation because each denomination is one *tenth* of the next larger denomination. Thus, units are *tenths* of tens, tens are *tenths* of hundreds, and hundreds are *tenths* of thousands.

Likewise *tenths* of units are written to the right of units, but separated from them by a *decimal point* (.). *Tenths* of tenths, or hundredths, are written in the next place to the right, and *tenths* of hundredths, or thousandths, in the next or third place from the decimal point. Thus, 324.516 consists of 3 hundreds, 2 tens, 4 units, 5 *tenths*, 1 *hundredth*, and 6 *thousandths*.

While the system is thus properly termed a system of decimal notation, the meaning of the word *decimal* is made more specific by restricting its use to the part of a number which is to the right of the decimal point; the whole numbers to the left being termed *integers*.

It will therefore be seen that the denominations to the right of units are the same as those to the left, except that they *decrease* by tens while those to the left *increase* by tens. These decreasing denominations are distinguished by adding the suffix *ths* to each denomination.

17. Reading and Writing Decimals. Beginning at the decimal point, decimals are *read* exactly as whole numbers are read, with the denomination of the last place to the right named. The name of the denomination of the right-hand

place is the same as if it were a whole number, with one *additional* place, and with the suffix *ths* added. The word *and* is always used between the whole number and the decimal. Thus, 3,204.6103 is read: three thousand two hundred four *and* six thousand one hundred three *ten-thousandths*.

Write decimals exactly as whole numbers are written, and then set off to the right of the decimal point the number of places indicated by the name of the right-hand denomination. The number of places to be set off is always *one less than the number of places required to write the corresponding denomination as a whole number*. This is because the units place is always to the left, and never to the right, of the decimal point. Thus, 506 ten-thousandths is written with *four* places (.0506) because ten thousand as a whole number requires *five* places. Ciphers are used to the left of the number to be written as a decimal when necessary to set it away from the decimal point. In whole numbers the decimal point is presumed to be at the right of units figure, whether it is written or not.

WRITE:

1. Sixty-eight thousandths.

2. Five tenths. Seven hundredths. Eighteen hundredths.

3. Six thousandths. Five hundred three thousandths. Eighteen thousandths. Seven hundred thirty-five ten-thousandths. Eighty-four hundred-thousandths.

4. Eighteen hundredths. Thirteen hundred-thousandths. Sixty-four tenths.

5. Eighty-five ten-thousandths. Three hundred twenty-three and fifty-six hundredths. Seventy-eight thousandths. Three hundred twenty-three thousandths.

READ:

1. .5, .25, .025, .0625, .0005.
2. .00001, .3762536, .00875, .000025.
3. 2.5, 25.25, 25.025, 700.07, .303.
4. .193, 4.2, 6.3028, .03275, 176.4.

18. Addition and Subtraction of Decimals. The processes and principles applied in the addition and subtraction of decimals are the same as of integers. The same care should be exercised in keeping like denominations in the same column. The decimal point in the sum or difference is placed directly under the decimal points in the numbers added or subtracted.

All decimals smaller than thousandths are usually dropped, one being usually added to the thousandths column when the discarded decimal is five tenths or more.

PROBLEMS

1.	2.	3.	4.	5.
7.17	6.789	23.768	6.34	37.68
.9	.075	.45	.0074	1.045
.0006	74.8	6.985	43.325	174.632
4.76	.4	17.005	24.18	35.986
5.0017	24.986	3.178	3.457	1.706

6. .024 + 1.54 + 74.6 + 27.878 = ?

7. 234.96 + 756 + 40.5 + 6.03 + 1.005 = ?

8. 2.054 + 35.78 + .067 + .65 + .268 = ?

9. .9 + 13.564 + 234.96 + 8.5 + .306 + 41.87 = ?

10. .335 + 23.75 + 601.76 + .007 + 35.86 = ?

11. .91 + 13.564 + 234.96 + 8.5 + .306 + 41.87 = ?

12. 1 − .063 = ? 13.435 − .106 = ?

13. 3.5872 + 1.2834 = ?

14. 6 − 2.763 = ?

15. 8 − 3.234 = ?

16. 4.1 + 67.5 + 42.007 + 17.14 + .0009 = ?

17. 34.006 − 15.556 = ?

18. 68.215 − 36.5 = ?

19. 94.35 − 36.7 = ?

20. 46.235 − 22.065 = ?

21. $216.745 - 176.89 = ?$

22. $681.34 - 95.275 = ?$

23. $14.367 + 743.65 + .8 + .306 + 9.845 + 834 + 7.63 = ?$

24. $3.8 + 1.576 + 3.42 + 4 + 2.372 + .8 + 354 = ?$

25. $43.382 - 17.06785 = ?$

19. Multiplication and Division by 1 with Ciphers Annexed. Since the value of each place in both whole numbers and decimals is one tenth of that to the left of it, any way by which each figure can be shifted one place to the *right*, or one place nearer units, will decrease its value to one tenth of its former value. Shifting two places would likewise decrease it to one hundredth, three places to one thousandth, etc. In the same way shifting one place to the *left* would multiply it by ten, two places by one hundred, etc. This may be accomplished by moving the decimal point to the left or right. Thus, $25 \div 10 = 2.5$ or $25 \times 10 = 250$, and $25 \div 1000 = .025$ or $.25 \times 1000 = 250$.

In multiplying by 1 with ciphers annexed, the decimal point is moved to the right as many places as there are ciphers in the multiplier. Thus,

$$326.02 \times 100 = 32602,$$

and

$$100.2437 \times 1000 = 100243.7.$$

In dividing by 1 with ciphers annexed, the decimal place is moved as many places to the left as there are ciphers in the divisor. Thus,

$$326.02 \div 100 = 3.2602,$$

and

$$3.42 \div 100 = .0342.$$

These simple facts are of great convenience in performing operations with decimals and in shortening multiplication and division of whole numbers.

NOTE. — Problems like those above should be given until the pupil thinks of no other way of multiplying or dividing by 10, 100, etc., than by

removing the decimal point the proper number of places to the right or left.

20. Multiplication of Decimals.

(*a*) 9 times .25 = 2.25.

When a decimal is multiplied by a whole number, the denomination of the decimal of the product is the *same* as that of the multiplicand.

(*b*) .1 times 9 means one tenth of 9 or .9 (Sec. 19).

$$.25 \times .1 = .025.$$

Multiplying by .1 is the same as dividing by 10; by .01 the same as dividing by 100, etc. (Sec. 19).

(*c*) $.25 \times .7 = ?$
$.25 \times .1 = .025$
$.025 \times 7 = .175$

(*d*) $.25 \times 6.25 = ?$
$.25 \times .01 = .0025$ (see *b*)
$.0025 \times 625 = 1.5625$ (see *a*)

In multiplying decimals by tenths, hundredths, etc., one tenth, one hundredth, etc., is found, and the result is then multiplied by the number of tenths, hundredths, etc., in the multiplier. Or,

Remove the decimal place in the multiplicand as many places to the left as there are decimal places in the multiplier, then multiply by the multiplier as a whole number.

PROBLEMS

1. 214.76×89.104
2. 3.0046×43.25
3. $.8756 \times .173$
4. $.045 \times 18$
5. $64 \times .032$
6. 24.075×16
7. 45.009×78
8. 50.13×4.321
9. 176.84×4.321
10. 95.817×1000

Find the cost of:

11. 24,800 bricks @ \$7.35 per M.
12. 875 lb. hay @ \$1.25 per hundredweight.
13. 186 bu. wheat @ 67½¢ per bushel.

14. 15,680 lemons at 65¢ per hundredweight.

15. 357 bu. oats @ 30½¢ per bushel.

16. 75 bbl. flour @ $4.15 per barrel.

17. 70 bbl. mess pork @ $10.50 per barrel.

18. 14 bbl. beef @ $14.40 per barrel.

19. 5 cases shredded codfish @ $4.90 a case.

20. 3 cases canned pineapples, 6 doz. each, @ $2.87½ per dozen.

21. 30 bbl. mess beef @ $14.85 per barrel.

21. Division when the Divisor is an Integer.

SOLVE:

1. .6 ÷ 2 = ? Just as 6 bu. ÷ 2 = 3 bu. (½ of 6 bu.), so 6 tenths ÷ 2 = 3 tenths or .6 ÷ 2 = .3.

2. 2.6 ÷ 2 = ? 2.6 ÷ 2 = 1.3. $2\overline{)2.6}$ → 1.3

3. .396 ÷ 3 **5.** 6.6 ÷ 4 **7.** .7236 ÷ 9 **9.** .16 ÷ 5

4. 12.15 ÷ 3 **6.** 3.68 ÷ 4 **8.** .084 ÷ 4 **10.** .54 ÷ 6

In division the decimal point is written in the quotient when it is reached in the dividend. In short division it should be placed directly below the decimal point of the dividend; in long division directly above.

ILLUSTRATIONS.

(*A*) $5\overline{).18}$ → .036 (*B*) $15\overline{).09}$ → .006 (*C*) $125\overline{)7.50}$ → .06, 7.50 (*D*) $26\overline{)32.24}$ → 1.24, 62, 104 (*E*) $24\overline{)1.728}$ → .072, 48

From inspection of problems *A*, *B*, and *E*, just above, it will be seen that *each figure in the dividend, to the right of the decimal point, requires a figure (or cipher) in the quotient.*

Ciphers may be added at will to the right of a decimal without altering the value. They are, therefore, annexed where necessary to permit further division, but in practice they are carried in the mind only and not written.

In problems *D* and *E*, the partial products are omitted, the subtraction being performed mentally (Sec. 15).

SOLVE:

1. 48.24 ÷ 3	**10.** 1.5 ÷ 4	**19.** 174.9 ÷ 75
2. 8.64 ÷ 4	**11.** .06 ÷ 15	**20.** 43.58 ÷ 671
3. .465 ÷ 3	**12.** 8.06 ÷ 5	**21.** 345.9 ÷ 329
4. 8.4 ÷ 5	**13.** .2 ÷ 8	**22.** 56.89 ÷ 137
5. .648 ÷ 6	**14.** 34.75 ÷ 25	**23.** .0789 ÷ 703
6. .114 ÷ 7	**15.** .0543 ÷ 15	**24.** .6789 ÷ 212
7. .81 ÷ 9	**16.** .0255 ÷ 11	**25.** 17.68 ÷ 245
8. .63 ÷ 9	**17.** 3.184 ÷ 482	**26.** 334.4 ÷ 76
9. .12 ÷ 9	**18.** 37.86 ÷ 541	

22. When the Divisor contains a Decimal.

EXAMPLE: $1.728 \div .12 = ?$

If we could change the divisor .12 to a whole number and solve as in Sec. 21, the problem of the location of the decimal point in the quotient (the only way in which division of decimals differs from division of whole numbers), would be an easy one.

But dividing by one tenth is equivalent to multiplying by ten, dividing by one hundredth to multiplying by one hundred, etc. (Sec. 20). Thus dividing 1.728 by .01 = 1.728×100 or 172.8 (Sec. 19), and dividing by .12 would give $\frac{1}{12}$ of the result obtained through dividing by .01. Thus:

$$1.728 \div .01 = 172.8,$$

and

$$172.8 \div 12 = 14.4.$$

In division by a decimal, therefore, *the decimal point of the divisor should be moved to the right of the last digit, and that of the dividend an equal number of places to the right of its original position* (Sec. 19). Division should then be performed as in Sec. 21, marking the decimal point in the quotient when the decimal point is reached in the dividend.

To avoid losing the identity of the original divisor and to be able to fix the exact remainder, should there be one, the position of the new decimal point should be indicated only. A small St. Andrew's cross (×) forms a convenient mark for that purpose.

SOLVE:

1. 6.336 ÷ 1.44.

$$\begin{array}{r} 4.4 \\ 1.44_{\times})6.33_{\times}6 \\ \underline{5\ 76} \\ 57\ 6 \\ \underline{57\ 6} \end{array}$$

2. 7.345 ÷ .29.
3. 250.754 ÷ 6.17.
4. 6.0534 ÷ 19.23.
5. 132.5 ÷ 734.
6. 784 ÷ 4.235.
7. .3416 ÷ .0189.
8. 12.347 ÷ .0074.
9. .8765 ÷ 3.422.
10. 9 ÷ 102.
11. 14.3768 ÷ .9817.
12. 84.45 ÷ .089.
13. 3894.78 ÷ 4287.
14. 346.543 ÷ 634.08.
15. 34.25 ÷ 84.6.
16. 9.1342 ÷ 208.4.
17. 1885 ÷ 28.47.
18. 363.71 ÷ 1.126.
19. 83.078 ÷ 3.57.
20. 137.854 ÷ .425.
21. 38.9007 ÷ .425.
22. 568.148 ÷ 201.03.
23. .81769 ÷ .0008175.
24. \$135 ÷ \$.37½.
25. 74 ÷ .0136.
26. .33614 ÷ 13.45.
27. 18.3467 ÷ 1.233.

PROBLEMS

1. Add eight and nine tenths; seven hundred twenty-six and twenty-five hundredths; one hundred sixty-eight and ninety-seven hundredths; one thousand three and seven tenths; seven hundred sixty-eight and seventy-four hundredths.
2. 967.45 + 8.674 + 23.997 + 864.325 + 37286 + 42.1 + 6.5 = ?
3. From six thousand seven take one thousand two hundred twenty-eight and seven hundredths.
4. What is the sum of 3.25, 1.8, 67.89, .0032, 879.435, and 23.067?
5. What is the sum of .125, 1.25, 12.5, 125, .0125?
6. From 675. take 67.893.
7. From 345.6703 take 43.52.
8. Multiply 54.054 by .0678.
9. Multiply 543.002 by 18.6.
10. Divide 6423.38 by 28.87.
11. Divide .00684 by .25.
12. At \$9.25 per ton, how much coal can be bought for \$67.53?
13. If a barrel of apples cost \$5.15, how many can be bought for 75?

14. At $.26 per dozen, how many eggs can be bought for $185.32?

15. At $6.45 per ton, how many tons of soft coal can be bought for $175?

16. A farmer sold 65 bu. wheat at $.62½, 34 bu. rye at $.58¼, 78 bbl. of apples at $6.40. He bought 35 lb. sugar at $.06, 25 gal. molasses at $.85, and a set of harness at $16.75. How much money had he left?

17. A carpenter earned $15.60 a week for 8 weeks. The first week he spent $8.75, the second week $11.45, and the remaining weeks he spent $8.50 per week on an average. How much money had he at the end of the time?

18. A man who has an income of $6785 per year spends $1385.75. How much does he save?

19. A hardware merchant had, at the beginning of the year, $9800. During the year he bought goods to the amount of $7845, and sold to the amount of $7856. If the goods he had on hand at the end of the year were worth $8340.65, what did he gain or lose during the year?

20. A man bought a mower for $67, a wagon for $56.50, a plow for $6, and a rake for $1.75. If he gave the merchant two one-hundred-dollar bills, how much change did he receive?

21. Mr. A. W. Springer bought of C. E. Dunlap, 75 bbl. flour at $5.14, and 34 bbl. buckwheat flour at $3.95. What was the amount of the bill?

22. Mr. S. H. Detmore bought of Wm. King, 124 boxes oranges at $1.12½, 12 boxes figs at $1.62½, 33 boxes apricots at $2.15, 15 boxes citrons at $1.33⅓, and 12 boxes layer raisins at $2.95. Find the amount of the bill.

23. How many cases tomatoes, 70 doz. to the case, at 92¢ per dozen, can be bought for $277.55?

Factors may, themselves, be either prime or composite.

A number which will exactly divide two or more numbers is said to be a *common divisor* or a *common factor*. The largest number which will exactly divide two or more numbers is their *greatest common factor* or *greatest common divisor*. If two or more numbers have no common divisor, they are said to be *relatively prime*.

A number which is two or more times another number is its *multiple*. When a number is a multiple of two or more numbers, it is their *common multiple*. When it is the smallest number which is a multiple of two or more numbers, it is their *least common multiple*.

27. Averaging. By *average* is meant the size of a part if the sum of a given series of numbers is distributed into equal parts. Thus, we say the average cost, average monthly expenses, average daily attendance, the assessed valuation *per capita*, etc.

In business, we may more clearly judge given conditions by the use of an average cost, output, etc., than if we were compelled to keep in mind the several amounts. In administrative statistics a more intelligible summary of facts may often be made by using averages. In many similar ways averaging enters into business, and it should be early understood.

PROBLEM. — A merchant's receipts for one week were: $140.45, $217.20, $200, $209.80, $432.75, and $630.40. Find his average daily receipts.

SOLUTION. — $140.45
217.20
200.00
209.80
432.75
630.40
$\frac{1}{6}$ of $1830.60 = $305.10, average daily receipts.

The general method of averaging seen in the above is to find the sum of the quantities to be averaged, and divide that sum into as many equal parts as there are quantities added.

PROBLEMS

1. If a man's gains for the year were as follows: $628, $75, $220, $865, $2205, $3600, $1780, $1500, $1240, $3275, $825, $775, what was his average monthly gain?

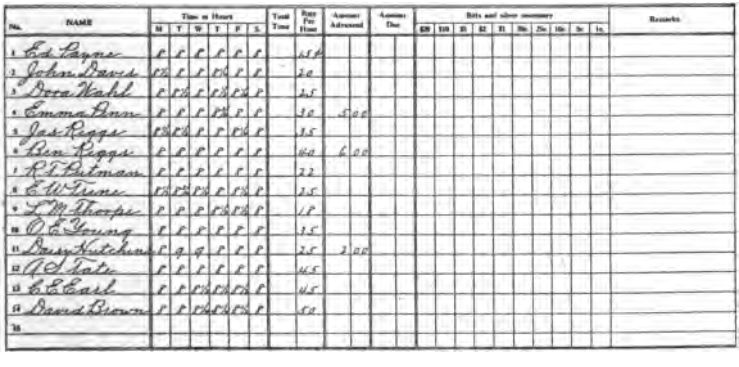
2. A grain dealer bought during the week: 2600, 3850, 4506, 7870, 9675, and 5490 bushels of wheat. What was the average daily purchase? He paid the market prices each day, which were quoted at 85¢, 85½¢, 88¢, 90¢, 84¢, and 80¢. What was the average cost per bushel? How would he know what price per bushel to ask that he may not lose in the transaction?

3. An agent's expenses were as follows: January, $125, February, $75, March, $80, April, $95, May, $105, and June, $225. What were his average monthly expenses?

4. Find the total time, the amount due, and the average daily wages for each laborer in the following time sheet, counting 8 hrs. to the day; also find the average wage for all.

Time Sheet for Week Ending April 30, 19—.

PAY ROLL



5. A student's grades for his high school course were as follows: Algebra 92, Geometry 86, Beginning Latin 89, Cæsar 94, Cicero 94,

Virgil 92, German 90, Literature 83, Essays 85, Chemistry 92, Physics 90, Physiology 95, Greek History 87, Roman History 88, U. S. History 87, Arithmetic 85, Bookkeeping 90. What was his average grade for the course?

28. To find the Value of more than One Part. *Tenths.* (Sec. 19.) To find $\frac{5}{10}$ take $\frac{1}{2}$. To find any other number of tenths, find $\frac{1}{10}$ as in section 19, and multiply by the number of tenths desired.

SOLVE:

$\frac{5}{10}$ of 323,496, $\frac{3}{10}$ of 769,284, $\frac{8}{10}$ of 6789.40, $\frac{7}{10}$ of 81,145, $\frac{4}{10}$ of 72,910, $\frac{3}{10}$ of 432,561, $\frac{8}{10}$ of .63245, $\frac{9}{10}$ of 237,684.

Twentieths. ($\frac{1}{2}$ of 1 tenth.) $\frac{10}{20} = \frac{1}{2}$, $\frac{2}{20} = \frac{1}{10}$, etc. The simplest form of the fraction should be taken and that fraction of the number then found. Practice until student uses the simplest form at first sight.

SOLVE:

1. $\frac{8}{20}$ of 360
2. $\frac{12}{20}$ of 8.460
3. $\frac{4}{20}$ of 3750
4. $\frac{5}{20}$ of 7260
5. $\frac{13}{20}$ of 3240
6. $\frac{14}{20}$ of 7.380
7. $\frac{12}{20}$ of 960
8. $\frac{14}{20}$ of 490
9. $\frac{18}{20}$ of 4260
10. $\frac{6}{20}$ of 428
11. $\frac{18}{20}$ of 5.061
12. $\frac{8}{20}$ of 576

Fifths. $\frac{2}{5} = \frac{4}{10}$, $\frac{3}{5} = \frac{6}{10}$, and $\frac{4}{5} = \frac{8}{10}$.

Fourths. $\frac{2}{4} = \frac{1}{2}$. For $\frac{3}{4}$, subtract $\frac{1}{4}$.

Sixths. $\frac{3}{6} = \frac{1}{2}$, $\frac{2}{6} = \frac{1}{3}$, $\frac{4}{6} = \frac{2}{3}$.

Eighths. $\frac{4}{8} = \frac{1}{2}$, $\frac{2}{8} = \frac{1}{4}$, $\frac{6}{8} = \frac{3}{4}$, etc.

Twelfths. $\frac{6}{12} = \frac{1}{2}$, $\frac{3}{12} = \frac{1}{4}$, $\frac{4}{12} = \frac{1}{3}$, etc. For $\frac{11}{12}$ subtract $\frac{1}{12}$.

In general, use simpler equivalent fractions wherever possible. If there are none, find the value of one part and multiply by the number of parts desired.

SOLVE:

1. $\frac{9}{15}$ of 726
2. $\frac{12}{17}$ of 543
3. $\frac{3}{8}$ of 789
4. $\frac{7}{9}$ of 911
5. $\frac{5}{20}$ of 614
6. $\frac{8}{11}$ of 729
7. $\frac{3}{13}$ of 728
8. $\frac{4}{7}$ of 329
9. $\frac{5}{8}$ of 628

10. $\frac{3}{14}$ of 7028
11. $\frac{7}{14}$ of 7028
12. $\frac{4}{15}$ of 328.65
13. $\frac{3}{20}$ of 16,924
14. $\frac{9}{12}$ of 6724
15. $\frac{8}{18}$ of 72,365
16. $\frac{7}{14}$ of 98,642
17. $\frac{3}{16}$ of 48,689
18. $\frac{5}{17}$ of 45,186
19. $\frac{4}{19}$ of .3876
20. $\frac{6}{18}$ of 4263
21. $\frac{3}{8}$ of 67.908
22. $\frac{3}{4}$ of 7252
23. $\frac{3}{8}$ of 81.720
24. $\frac{4}{7}$ of 36,428
25. $\frac{3}{18}$ of 7829
26. $\frac{4}{10}$ of 1.7206
27. $\frac{3}{8}$ of 72,963
28. $\frac{4}{17}$ of 34,102
29. $\frac{4}{17}$ of 4291
30. $\frac{6}{14}$ of 28,367

SOLVE:

1. $\frac{14}{23}$ of 68,734
2. $\frac{15}{87}$ of 57,836
3. $\frac{41}{98}$ of 196,732
4. $\frac{37}{102}$ of 67.941
5. $\frac{23}{62}$ of 94,362
6. $\frac{15}{32}$ of 6946
7. $\frac{42}{371}$ of 198,674
8. $\frac{17}{25}$ of .4575
9. $\frac{45}{120}$ of 9.7176
10. $\frac{58}{275}$ of 32,642
11. $\frac{62}{172}$ of 19,643
12. $\frac{21}{96}$ of 42,782
13. $\frac{25}{62}$ of 32,785
14. $\frac{29}{85}$ of 42,363
15. $\frac{30}{75}$ of 89.642
16. $\frac{28}{640}$ of 2632
17. $\frac{15}{47}$ of 15,922
18. $\frac{76}{195}$ of 58,596
19. $\frac{98}{195}$ of 155,476
20. $\frac{38}{190}$ of 167.1918
21. $\frac{89}{367}$ of 47.6297
22. $\frac{30}{125}$ of 62,587
23. $\frac{78}{234}$ of 29,410
24. $\frac{18}{783}$ of 25,000
25. $\frac{15}{213}$ of 67,894
26. $\frac{120}{318}$ of .456781
27. $\frac{27}{81}$ of 86,543
28. $\frac{83}{96}$ of 219,876
29. $\frac{21}{87}$ of 98,634
30. $\frac{48}{128}$ of 186,792

29. Simplified Processes. It often happens that the numbers to be multiplied or divided are such that the operations of multiplication and division may be much shortened. The simplicity of the shortened method, too, often lends itself to clearer thinking and lessens the liability of error. The following suggestions are not given to be learned as "short methods," but as examples of what may be done to simplify problems if a little thinking is done before the pupil plunges into the mechanics of a solution. They should form a part of the methods habitually employed and not merely referred to when longer processes have been drilled into habit by much practice. A keen appreciation of the possibilities for shortening and simplifying many of the arithmetical processes will lead to quick work and accurate results.

ILLUSTRATIONS:

1. $426 \times 100 = ?$ By $1000 = ?$ By $10 = ?$ (Sec. 19.)

SUGGESTION. In multiplying by a number consisting of 1 with ciphers annexed, the product is found to consist of the multiplicand with the ciphers annexed. Thus, $728 \times 1000 = 728{,}000$.

2. $53{,}648 \div 1000 = ?$ (Sec. 19.)

Since the divisor will take out even thousands, the hundreds, tens, and units figures will form the remainder. Thus, $53{,}648 \div 1000 = 53$ and $\frac{648}{1000}$, or 53.648. Likewise, $8269 \div 100 = 82$ and $\frac{69}{100}$, or 82.69.

3. $7268 \times 25 = ?$

Multiplying by 25 would give $\frac{1}{4}$ as much as multiplying by 100. Thus, the above may be written at sight as 182,300 ($\frac{1}{4}$ of 726,800). Likewise $7268 \times 50 = \frac{1}{2}$ of 726,800 or 363,400.

4. $742 \div 25 = ?$

The quotient is manifestly 4 times what it would be if 100 were the divisor. In practice, multiply first and divide by 100. Thus, $742 \times 4 = 2968$, which divided by $100 = 29$ and $\frac{68}{100}$, or 29.68. Likewise $742 \div 50 = 7.42 \times 2$ or 14.84.

5. $246 \times 12\frac{1}{2} = ?$ ($\frac{1}{8}$ of 24,600.)

6. $368 \div 12\frac{1}{2} = ?$ ($368 \times 8 \div 100$.)

7. $1284 \times 15 = ?$

If the table of 15's has not been learned, the usual process may be shortened by annexing a cipher and adding $\frac{1}{2}$ of the number thus formed to itself. Thus,

$$\begin{array}{r} 12{,}840 \\ 6420 \\ \hline 19{,}260 \end{array}$$

8. Divide 5286 by 20. By 30.

The quotient, dividing by 20, is clearly $\frac{1}{2}$ as much as when divided by 10. Thus, $\frac{1}{2}$ of $528.6 = 264.3$.

9. $8246 \times 98 = ?$

It is evident that the product would be 100 times the multiplicand less twice the latter. Thus,

$$\begin{array}{r} 824{,}600 \\ 16{,}492 \\ \hline 808{,}108 \end{array}$$

10. To multiply by 11 or any multiple of 11.

$$328 \times 11 = ? \quad 476 \times 66 = ?$$

To multiply by 11 is to multiply by $10 + 1$. Therefore, multiply 328 by 10 and add 328; or add the digits as follows: 8; $8 + 2 = 10$; $2 + 3 + 1$ (carried) $= 6$; bring down 3. The result is 3608.

66 is 11 times 6. Multiply 476 by 11, and that by 6. It may be done as follows: $6 \times 6 = 36$; write 6 and carry 3. $6 + 7 = 13$; $6 \times 13 + 3$ (carried) $= 81$; write 1 and carry 8. $7 + 4 = 11$; $6 \times 11 + 8$ (carried) $= 74$; write 4 and carry 7. $6 \times 4 + 7$ (carried) $= 31$; write 31. The result is 31,416.

11. When one part of the multiplier is contained in another, the multiplication may be shortened as shown in the following:

$$342 \times 248 = ? \quad 167 \times 412 = ?$$

Since 24 is 3 times 8, it is evident that if 342 be multiplied by 8 and that result by 3, the final result will be the same as though we multiplied 342 by 248 in the usual manner (*a*).

167 multiplied by 4 gives 668. 668 multiplied by 3 $(12 + 4)$ gives 2004. The final result is the sum of 668 written in hundreds' place and 2004 in units' place, or 68,804 (*b*).

(*a*)	(*b*)
342	167
248	412
2,736	668
82,08	2004
84,816	68,804

12. $52 \times 225 = ?$

In the multiplier 225, we see twice and $\frac{1}{4}$ of 100 times 52, or

$$\begin{array}{r} 10{,}400 \\ 1{,}300 \\ \hline 11{,}700 \end{array}$$

13. $7558 \times 125 = ?$ $\frac{1}{8}$ of (7558×1000).

14. $57{,}632 \div 370 = 5763.2 \div 37$.

SOLVE:

1. $37{,}685 \times 10$	11. $32{,}764 \times 12\frac{1}{2}$	21. $16{,}324 \times 95$
2. $49{,}652 \times 100$	12. $25{,}670 \times 742$	22. $17{,}264 \times 320$
3. $29{,}627 \times 1000$	13. $62{,}482 \times 250$	23. $98{,}643 \div 20$
4. $72{,}864 \times 25$	14. $41{,}628 \times 75$	24. $76{,}523 \times 300$
5. $22{,}345 \times 15$	15. $29{,}863 \times 99$	25. $8629 \div 12\frac{1}{2}$
6. $28{,}364 \times 125$	16. $36{,}486 \times 175$	26. $37{,}520 \div 25$
7. $48{,}627 \times 55$	17. $28{,}634 \times 325$	27. 4286×88
8. $26{,}327 \times 98$	18. $67{,}450 \times 217$	28. $36{,}742 \times 97$
9. 4652×225	19. 6284×1500	29. $75{,}267 \times 420$
10. $63{,}245 \times 30$	20. 729×40	30. $2672 \div 50$

30. Decimal Equivalents of the More Common Fractional Parts.

SOLVE:

1. $\frac{1}{5}$ = how many tenths?
   $\frac{1}{5} = \frac{1}{5}$ of 10 tenths = 2 tenths or .2.
2. $\frac{1}{4}$ = how many hundredths?
   $\frac{1}{4} = \frac{1}{4}$ of 100 hundredths or .25.
3. $\frac{1}{8}$ = how many thousandths?
   $\frac{1}{8} = \frac{1}{8}$ of 1000 thousandths or .125.
4. $\frac{1}{2}$ = how many tenths? How many hundredths?
5. $\frac{1}{10}$ = how many tenths? How many hundredths?
6. $\frac{1}{3}$ = how many tenths? How many hundredths? How many thousandths?
   $\frac{1}{3} = \frac{1}{3}$ of ten tenths or $.3\frac{1}{3}$.
7. $\frac{1}{8}$ = how many tenths? How many hundredths? How many thousandths?

8. $\frac{1}{7}$ = how many tenths? How many hundredths? How many thousandths?

9. $\frac{1}{9}$ = how many tenths? How many hundredths? How many thousandths?

10. $\frac{1}{12}$ = how many tenths? How many hundredths? How many thousandths?

11. $\frac{1}{20}$ = how many tenths? How many hundredths? How many thousandths?

12. $\frac{1}{25}$ = how many tenths? How many hundredths? How many thousandths?

Table of decimal equivalents. Memorize.

$\frac{1}{2} = .5$	$\frac{1}{4} = .25$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{1}{12} = .08\frac{1}{3}$
$\frac{1}{5} = .2$	$\frac{1}{8} = .125$	$\frac{1}{7} = .14\frac{2}{7}$	$\frac{1}{20} = .05$
$\frac{1}{10} = .1$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{9} = .11\frac{1}{9}$	$\frac{1}{25} = .04$

NAME DECIMAL EQUIVALENTS AT SIGHT:

1. $\frac{3}{4}$	**5.** $\frac{7}{8}$	**9.** $\frac{8}{10}$	**13.** $\frac{3}{7}$	**17.** $\frac{5}{9}$
2. $\frac{2}{5}$	**6.** $\frac{4}{5}$	**10.** $\frac{5}{8}$	**14.** $\frac{4}{7}$	**18.** $\frac{4}{9}$
3. $\frac{3}{5}$	**7.** $\frac{2}{7}$	**11.** $\frac{11}{12}$	**15.** $\frac{5}{7}$	**19.** $\frac{7}{12}$
4. $\frac{3}{8}$	**8.** $\frac{2}{5}$	**12.** $\frac{5}{6}$	**16.** $\frac{6}{7}$	**20.** $\frac{5}{9}$

31. Fractional Parts of One Dollar.

The more common fractional parts of \$1 are:

$50¢ = \$\frac{1}{2}$	$10¢ = \$\frac{1}{10}$	$12\frac{1}{2}¢ = \$\frac{1}{8}$	$6\frac{2}{3}¢ = \$\frac{1}{15}$
$25¢ = \$\frac{1}{4}$	$33\frac{1}{3}¢ = \$\frac{1}{3}$	$8\frac{1}{3}¢ = \$\frac{1}{12}$	$3\frac{1}{3}¢ = \$\frac{1}{30}$
$20¢ = \$\frac{1}{5}$	$16\frac{2}{3}¢ = \$\frac{1}{6}$	$6\frac{1}{4}¢ = \$\frac{1}{16}$	$2\frac{1}{2}¢ = \$\frac{1}{40}$

The more usable multiples of these parts are:

$37\frac{1}{2}¢ = \$\frac{3}{8}$	$87\frac{1}{2}¢ = \$\frac{7}{8}$	$75¢ = \$\frac{3}{4}$	$60¢ = \$\frac{3}{5}$
$62\frac{1}{2}¢ = \$\frac{5}{8}$	$66\frac{2}{3}¢ = \$\frac{2}{3}$	$40¢ = \$\frac{2}{5}$	$80¢ = \$\frac{4}{5}$

32. Finding the Cost of Articles. A large part of the multiplication in business consists in finding the cost of articles at a given price each or per dozen. As prices are in a large number of cases a simple fractional part of one

dollar, this fact may be used to shorten the work, and what is of greater importance, to secure greater accuracy in result. Thus,

(*a*) Find cost of 344 yd. of calico at 12½¢ per yard. At 12½¢ it would cost ⅛ as much as it would at $1. ⅛ of $344 = $43.

NOTE. — In problems like this where the multiplicand is an easier number to multiply by than the real multiplier, it is so used, keeping in mind the denomination of the product.

(*b*) Paid $403 for corn at 20¢ per bushel. How many bushels did I buy?

$$403 \text{ bu.} \times 5 = 2015 \text{ bu.}$$

The number of bushels bought is clearly 5 times greater than if it were worth $1 per bushel.

FIND COST OF:

1. 7286 bu. wheat @ 50¢.
2. 1456 yd. prints @ 12½¢.
3. 764 yd. cloth @ 33⅓¢.
4. 324 bbl. mess beef @ $15.
5. 750 bbl. pork @ $15.12½.
6. 24 doz. cans tomatoes at $1.12½.
7. 90 doz. cans peas @ $1.37½.
8. 35 boxes figs @ $1.62½.
9. 1491 lb. tea @ 33⅓¢.
10. 450 sugar-cured hams, 5400 lb., @ 12½¢.
11. 348 boxes oranges @ $1.75.
12. 117 bbl. flour @ $6.75.
13. 16 hams, 195 lb., @ 16⅔¢.
14. 48 doz. cans of tomatoes @ 75¢.
15. 108 bbl. N.Y. buckwheat flour @ $4.25.
16. 125 doz. cans tomatoes @ $2.66⅔.
17. 96 doz. cans peas @ $1.37½.

18. 6 tierces refined lard, 2096 lb., @ 8¼¢.
19. 330 doz. jars of mustard @ 87½¢.
20. 368 lb. coffee @ 20¢.
21. 178 bbl. beef @ $20.

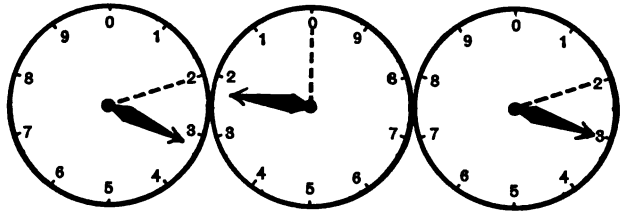
33. Cost of Goods Sold by the Hundred. Freight tariffs are usually based on one hundred pounds. Live stock is quoted by the hundredweight, and many other articles are sold in lots of one hundred. Since cents are hundredths of a dollar, *goods sold by the hundred will cost as many cents per unit as dollars per hundred.* Thus $5 per cwt. is 5¢ per pound, and $3.75 per C is $.0375 per pound.

FIND COST OF:

1. Freight charges on 4230 lb. of merchandise at $1.40 per cwt.
   $.014 × 4230 = $59.22. ($4.23 × 14 = $59.22.)
2. 1280 lb. nails @ 32¢ per cwt.
3. 2963 lb. rock salt @ $1.20 per cwt.
4. 2974 lb. fence wire @ $4.50 per cwt.
5. 1280 posts, split, @ $18 per C.
6. 375 lb. lead @ $4.75 per cwt.
7. 5 cattle, averaging 925 lb., @ $4.50 per cwt.
8. 11,580 posts, round, @ $25 per C.
9. 9850 lb. pork @ $7.50 per cwt.
10. Freight on a carload of grain, 46,500 lb., @ 38¢ per cwt.
11. 1378 lb. poultry @ $6.50 per cwt.
12. 1865 lb. beef @ $14 per cwt.
13. 975 lb. bran @ 90¢ per cwt.

34. Goods Sold by the Thousand. Brick, lumber, shingles, and many other articles are sold by the thousand. Gas is sold by the thousand cubic feet. The amount consumed is shown by a meter, upon the face of which are usually three dials. The dial to the right shows hundreds of cubic feet, that in the middle shows thousands, and that to the left

shows tens of thousands. Reading the last figure passed by the pointers, and going from left dial to right, the number of hundreds of cubic feet consumed is shown. Thus, the accompanying illustration reads 323 hundreds or 32,300 cubic



A Gas Meter

feet. The reading for the preceding period is shown by the dotted lines. The reading for the preceding period is subtracted from the present reading to find amount of gas consumed.

A mill being $\frac{1}{1000}$ of a dollar, *articles sold by the thousand are as many mills per article as dollars per thousand.* Thus, $37 per M is 37 mills per pound or foot, or $26.50 per M is 26.5 mills ($.0265) each.

Find Cost of:

1. 1625 ft. oak lumber @ $36 per M.

One foot would cost 36 mills ($.036), and 1625 ft. would cost $.036 × 1625 ($1.625 × 36) or $58.50.

2. 20,408 ft. flooring @ $17.50 per M.

3. 14,450 ft. 2 × 6–15 @ $15.50 per M.

4. 10,458 ft. 6 × 6–18 @ $22.50 per M.

5. 10,448 ft. sheathing @ $12.75 per M.

6. 612 ft. pine lumber @ $15 per M.

7. 456 ft. spruce @ $12.50 per M.

8. 7750 shingles @ $5.25 per M.

9. The meter readings for gas consumed in a residence for six months were as follows: Sept. 1, 28,400; Oct. 1, 31,000; Nov. 1, 34,400; Dec. 1, 37,600; Jan. 1, 40,700; Feb. 1, 43,200; Mar. 1, 45,100.

At 90¢ a thousand, what was the total of the monthly gas bills for the six months?

10. A contractor bought material for a building as follows: 567,800 brick @ $6.75; 35,657 ft. matched pine @ $20; 14,720 ft. hemlock @ $12; 4680 ft. walnut @ $45; 75,250 shingles @ $4.75. What was the total cost?

11. If there is a gas meter in the school building, read it from month to month and compute the cost of gas. Also read the meters in your home and compute the cost of gas.

35. Goods Sold by the Ton. Coal, hay, and other articles are sold by the ton. If coal is quoted at $6.50 per ton, it is $3.25 per 1000 pounds, and 3.25 mills ($.00325) per pound. In other words, *goods sold by the ton are one-half as many mills per pound as dollars per ton.* Thus, $3.80 per ton is 1.9 mill ($.0019) per pound.

PROBLEMS

1. 3640 lb. hay @ $8.50 per T.

½ of $8.50 = $4.25 per M., or 4.25 mills per lb., $.00425 × 3640 = $15.47.

2. 42,300 lb. hay @ $9 per T.

3. 2860 lb. soft coal @ $3.75 per T.

4. 84,375 lb. steel rails @ $20 per T.

5. 225,780 lb. ore @ $25 per T.

6. 3500 lb. fertilizer @ $22 per T.

7. 4525 lb. anthracite coal @ $8.75 per T.

8. 38,960 lb. salt @ $3.35 per T.

9. 265,700 lb. clover hay @ $6.20 per T.

10. What will be the cost of the freight on 5 cars of coal at $2.75 per ton, the cars weighing respectively: 87,560, 75,605, 54,780, 85,670, and 70,840 pounds net?

11. FIND THE VALUE OF EACH:

ARTICLE	GROSS WEIGHT	TARE	PRICE
A load of hay	3450 lb.	1256 lb.	$8.75 per T.
A load of straw	2975 lb.	856 lb.	$2.60 per T.
A load of coal	5475 lb.	1680 lb.	$7.50 per T.
A load of beets	3475 lb.	1240 lb.	$12.50 per T.
A load of stone	4250 lb.	1250 lb.	$14.50 per T.

36. Invoicing, Bills, and Accounts. When goods are shipped, or sold on account, an *invoice* or *bill* is mailed, or sent with the goods. A detailed statement of the amount, kind, and prices of the goods, together with the names of the parties to the transaction, term of credit, condition of sale, discount allowed, etc., is called either a *bill* or *invoice*. The term *bill* is applied particularly to a statement of goods bought, of services rendered, or of work performed. Promptness in billing goods shipped, and accuracy in computing the amount of the bills, are business essentials.

Formerly, the term "invoice" was used when goods were shipped on consignment only, but it is now often used interchangeably with "bill." A clause on the invoice, stating that the goods remain the property of the consignor until paid for, makes the sending them out a *consignment* rather than a sale.

At stated periods, usually the first of each month, a *statement* of account is sent to debtors. A statement merely gives the amount of bills previously rendered, and credits, with their dates. Its chief purpose is to remind the debtor of the debt, but it is also of assistance in checking errors in accounts.

PROBLEMS

Rule forms and copy the following invoices, filling in the missing amounts.

Philadelphia, Pa., Jan. 19, 19—

1. The Amos King Co.,
Baltimore, Md.

Bought of JOHN WANAMAKER

Terms: Net 30 days

1310	10	Roll Top Office Desks,	$43.75			
X 1338	15	Typewriter Desks,	4.50			
1317	20	Office Tables,	7.00			
1238	10	Office Chairs,	5.75			

Note. — The figures to the left of the ruling show the catalogue numbers.

2.

Pittsburg, Pa., May 1, 19—

M^r. Samuel C. Jackson,
Columbus, O.

In account with **A. D. HIRSCH & COMPANY**

		Dr.				
Apr.	1	To account rendered	37	50		
	10	" Mdse.	185	00		
	25	" "	356	75		
		Cr.				
	14	By cash	100	00		
	24	" 30-day note	300	00		

Chicago, Ill., May 6, 19—

3. Mr. S. A. Daubb,
Evanston, Ill.

Bought of _____

Terms: 30 da. 2% 10 da.

1690 ft. N.C. Ceiling	$18.50
20,165 ft. Flooring	28.50
3520 ft. Studding	10.80
12,500 Shingles	5.75

4. Make out bill and find the amount of the following invoice of goods, shipped to Messrs. Chase and Witherspoon, Richmond, Ind., by The Sprague Grocery Co., Chicago, Ill., May 1, 19—; terms: 60 da. net, 3% 10 da.: 20 hf. chests Japan Tea, 1200#, @ 23¢; 20 hf. chests Oolong Tea, 1000#, @ 48¢; 16 cases Ceylon Tea, 800#, @ 35¢; 10 bags Mocha Coffee, 1500#, @ 24¢; 20 bags Java Coffee, 1500#, @ 25¢.

Find the amount of the following invoices:

5. N. E. Nash & Co., Chicago, sold Young & Hooper, Ft. Wayne, Ind., 135 bbl. Mess Pork @ $11.50; 40 bbl. Mess Beef @ $11.75; 30 tierces R'f'd Lard, 10,500#, @ 6½¢; 450 Sugar-cured Hams, 1500#, @ 12½¢.

6. A. D. Cunningham bought of C. E. Kinsley on account 30 da., 24 pr. Congress Shoes @ $1.60; 25 pr. Men's Heavy Shoes @ $1.75; 22 pr. Ladies' French Kid Shoes @ $3.25; 12 pr. Boys' Shoes @ $.80; 14 pr. Infants' Shoes @ $.90; 22 pr. Calf Boots @ $2.25. Make out the bill, supplying dates and places, and take off $\frac{1}{5}$ for cash.

7. 5 Brass Bedsteads No. 200 @ $30; 6 rolls Wilton Carpet, 280 yd., @ 80¢; 4 Hair Mattresses, Style BB, @ $12.50; 16 rolls Ingrain Carpet, 1256 yd., @ 40¢; 8 rolls Moquette, 432 yd., @ 85¢; 12 pr. Portieres @ $8; 5 Antique Oak Bedsteads @ $15.20; discount $\frac{1}{10}$.

8.

New York, N. Y., Feb 20, 19—

Mrs C. W. Lane

Reading, Pa.

Bought of **R. H. MACY & COMPANY**

Terms: Cash 255 Broadway

5	Star Hams	72 lb. 23¢	16	56	
2	Mats Java Coffee	150 " 25¢	37	50	
2	cases Tomatoes	4 doz. 1.75	7	20	
3	bbl. Gold Medal Flour	6.50	19	50	
	Received Payment				
	R. H. Macy & Co.				
	per C. W. M.				

Render the following statements:

9. Jan. 31, 19—, the debits and credits of John C. Scott in account with the Simmons Hardware Co., of St. Louis, Mo., were as follows:

Debits: Jan. 1, To account rendered, $157.25
Jan. 7, To Mdse. $32
Jan. 12, To Mdse. $345

Credits: Jan. 5, By cash, $150
Jan. 10, By 20-day note, $60

10. Oct. 31, 19—, the debits and credits of W. C. McClure with Lord & Taylor, New York City, were:

Debits: Oct. 1, To account rendered, $86.25
Oct. 10, To Mdse., $165
Oct. 19, To Mdse., $425
Oct. 29, To Mdse., $136.50

Credits: Oct. 5, By cash, $80
Oct. 20, By note for 60 days for balance due

11. On Nov. 30, 19—, the account of Wm. E. Bacon with American Prism Co., Cincinnati, O., was as follows:

Debits: Nov. 2, To Mdse., $109.60
Nov. 8, To Mdse., $67.80
Nov. 21, To Mdse., $240

Credits: Nov. 15, By cash, $80
Nov. 20, By 10-day note, $75
Nov. 29, By cash, $200

27. Pay Rolls.

PAY ROLL For the week ending _____ 19__

No.	NAME	Number of Hours Work Each Day						Total No. of Hours	Wages per Hour	Total Wages		Remarks
		M	T	W.	T.	F.	S.					
1	A. E. Brown	8	8	8	8	8	8	48	25¢	12	00	
2	C. E. Kuesters	8½	8	7½	8	8	8½	48½	30¢	14	55	
3	B. W. Wright	8	8	8	8½	8	8		24¢			
4	L. E. Goss	8	8	8	8	8	8		25¢			
5	W. D. Dunn	8	8	8	8	7½	8		35¢			
6	Chas. Good	8	8	8½	8	8	8		18½			
7	W. K. Martin	8	8	8½	8	8½	8		20¢			
8	J. P. Little	8	8	8	8	8½	8		25¢			
9	O. P. Hopper	8	8	9	8	8	8½		37½			
10	Jas. Sprague	8	8	9	8½	8	8		30¢			

Copy the above pay roll and fill out the blank spaces as shown in Nos. 1 and 2.

Some firms use checks in paying off employes, but many find it more convenient to pay in cash by the envelope system. To pay off men by the envelope system, it is necessary to find first the amount of money needed, then the bills and fractional currency needed. To do this, a blank called

a *change memorandum*, similar to the one shown below, is generally used.

Fill out the accompanying change memorandum for the pay roll given above. Make totals and check results.

No.	Bills					Coins				
	$20	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1		1		1		1			1	
2		1		2						
3										
4										
5										
6										
7										
8										
9										
10										

FIRST NATIONAL BANK
UNION CITY, MO.

Pay-roll Memorandum

Union City Mfg. Co. require the following:

Pennies . . .	8	.08
Nickles . . .	6	.30
Dimes	7	.70
Quarters . . .	8	2.00
Halves	6	3.00
Dollars . . .	4	4.00
2's	7	14.00
5's	3	15.00
10's	9	90.00
20's	1	20.00
		149.08

When the amount of the pay roll and the necessary bills and fractional currency have been determined, a *pay-roll memorandum*, similar to the accompanying form, is taken to the bank and the necessary amount of each denomination of money secured. The pay-roll memorandum should foot the same as the pay-roll book.

Make out a pay-roll memorandum for the preceding pay roll, using the accompanying form as a model.

PROBLEMS

1. A man worked 9 months, 26 days per month, and received $912.60. What was his daily wage?

2. I hold $2000 worth of stock in a company capitalized at $36,000. A dividend of $4200 is declared. What is my share?

3. I have $1200 on interest at 5%. What does it yield annually? ($5\% = \frac{1}{20}$.)

4. Load deliveries of coal at a school building for the month of February were as follows: 4500, 4650, 3850, 4000, 3675, 3800, 4200, 4350, 4500, 4100, 4350, 4400, 4050, 3900, 3600, 3550, and 3850 lb. net. Find the total delivery, the average weight per load, the total cost at $4.20 per ton, and the average cost per load.

5. A society, after raising $17,675 by subscription, erected a building that cost $22,500. How much did each of its 25 members have to pay on a per capita assessment?

6. In a woolen factory there are 48 looms. If in 208 days they wove 269,568 yd. of cloth, how much was woven on each loom? How much on all in one day? On each in one day?

7. A shoe dealer bought 35 cases of boots, containing 12 pairs each. For the lot he paid $2100. How much did they cost per case? Per pair?

8. The entire cost of constructing a railroad 25 miles long was $1,736,075. What was the cost per mile?

9. The population of Rhode Island in 1900 was 428,556. The area is 1250 sq. mi. What was the number of inhabitants to the square mile?

10. A schoolroom containing 72,500 cu. ft. of air is occupied by 125 pupils. How much air is that to each pupil? If the air is completely changed every 10 minutes, how much fresh air is provided for each pupil per hour?

11. A county containing 488,000 acres is divided into 27 townships of equal area. How many acres in each township?

12. The population of the United States in 1900 was 74,627,907. The number of congressmen was 357. What was the average number of persons represented by each congressman?

13. An agent earns monthly commission as follows: $75, $92, $83, $66, $55, $28, $82, $160, $32, $280, $175, $196, and $215. What is his average commission per month?

14. In one year the national banks of New York City loaned $236,327,598. What was the average weekly loan?

15. The total cost of building the Union Pacific Railway was $18,941,400. The road is 1177 miles long. What was the cost per mile?

16. A man bought a house and lot for $6107. He paid $1392 down and agreed to pay the balance in 115 monthly payments. What was the amount of each payment?

17. If a gas burner consumes 1100 cu. ft. of gas in 160 hours at a cost of $1.42 per thousand, what is the cost per hour? How many cubic feet does it burn in one hour?

18. How much will it cost to ship a carload of corn containing 45,000 lb. from Crete, Neb., to Chicago, if the freight rate is 6¢ per hundred pounds? What would be the cost per bushel of 56 lb.?

19. The average wages of a steel mill employing 4500 men is $3.50 per day. If a reduction of $\frac{1}{10}$ is made in wages, how much is the company's pay roll reduced?

20. Find the amount of the following invoice of goods:

10 pc. sateen, 55½, 51, 50¾, 52, 57, 54½, 50, 56½, 53½, 51 @ 6¼¢; 11 pc. flannel, 62½, 60, 65, 67½, 65, 64, 63½, 67, 78, 62¾, 63 @ 6⅞¢; 15 pc. gingham, 50, 51, 52, 54½, 56, 57, 58½, 54¾, 59, 60, 61, 56½, 52½, 53¾, 54 @ 12½¢.

21. Find the amount of the following pay roll (*a*) on the basis of an 8-hour day; (*b*) on the basis of a 10-hour day:

Name	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Rate
Lawrence Dodd	7	8	10	9	8	11	$3.00
Henry Wahl	8	8	8	8	8	10	2.50
James Ringo	6	7	5	8	9	1	3.50
Leroy Hines	9	6	8	10	12	12	3.00
Thomas Cox	8	8	8	9	9	10	1.75
Amos Zeering	9	9	9	9	9	8	2.50
Adam Frick	8	8	8	8	8	8	3.00
Warren Hastings	9	10	10	10	11	12	2.25
Gordon Bang	7	6	5	6	8	8	3.00
Emil Ehman	8	8	9	5	6	5	4.00

22. Make pay-roll memorandum and change memorandum for above problem.

V

FRACTIONS

38. Definitions. By *fraction* is meant one or more of the equal parts of some quantity. The size of a part is indicated by the *denominator* in showing the number of parts into which the whole is divided. Inasmuch as fourths and halves are as truly different things as bushels and pecks, it follows that the use of the denominator is to *denominate* or name the kind of parts. The number of parts used is indicated by the *numerator* (numberer). Thus, in $\frac{5}{8}$ and $\frac{3}{7}$, and in 5 bu. and 3 pk., the denominators 8 and 7 and the words bushels and pecks indicate the kind of parts or units, while 5 and 3, in each case, show the number of those units. The numerator and denominator are called the *terms* of the fraction.

By *common* or vulgar fractions are meant fractions of *ordinary* kind, *i.e.* fractions expressed by a numerator written above a line and a denominator written below the line. A common fraction whose denominator is 1, with a number of ciphers annexed, is called a *decimal* fraction, because its denominator expresses a decimal denomination (Sec. 16). Such a fraction may be written at pleasure as a decimal fraction or as a decimal. (Sec. 17.)

A fraction which represents a part of a quantity less than unity is called a *proper fraction;* if it is equal to or greater than unity, it is *improper*. The numerator of a proper fraction is numerically less than the denominator; the numerator of an improper fraction is equal to or greater than the denominator. A *mixed* number contains a whole number and a fraction.

Business computations require constant use of fractions. A practical knowledge of fractions, however, necessitates only a thorough mastery of operations with the simpler fractions, and skill in their use. Little attention need be given to fractions having larger terms.

39. Reduction of Fractions. The same fraction may be expressed in different denominations. Thus, $\frac{1}{2}$ may be written as $\frac{2}{4}$ or $\frac{3}{6}$.

A change in denomination requires a change in the denominator. If the value of the fraction is not to be changed, then the numerator must also be changed and in exactly the same ratio. In other words, *the terms of a fraction may be multiplied or divided by the same number without changing the value of the fraction.*

When the change results in terms of less numerical value, the fraction has been *simplified* or *reduced* to *lower terms.*

It should be remembered that one may add, subtract, or divide *like quantities only.* Three quarts may not be added to five pints until they are changed to the same denomination. Neither is it possible to add $\frac{3}{4}$ and $\frac{3}{8}$ or to find how many $\frac{3}{8}$'s there are in $\frac{3}{4}$ without reducing to like denomination. In division of fractions this step is obscured by the commonly used process of inversion, but it is nevertheless performed. The recognition of these facts and their constant application will do much to simplify the whole subject of fractions and give facility in computations involving them.

SIMPLIFY:

1. $\frac{12}{180}$	7. $\frac{144}{216}$	13. $\frac{98}{196}$	19. $\frac{52}{1000}$	25. $\frac{875}{1000}$
2. $\frac{88}{108}$	8. $\frac{50}{625}$	14. $\frac{152}{304}$	20. $\frac{144}{384}$	26. $\frac{184}{204}$
3. $\frac{72}{84}$	9. $\frac{42}{56}$	15. $\frac{78}{221}$	21. $\frac{152}{875}$	27. $\frac{576}{640}$
4. $\frac{275}{300}$	10. $\frac{51}{68}$	16. $\frac{192}{120}$	22. $\frac{875}{1000}$	28. $\frac{280}{324}$
5. $\frac{14}{42}$	11. $\frac{28}{60}$	17. $\frac{25}{100}$	23. $\frac{125}{1000}$	29. $\frac{112}{187}$
6. $\frac{78}{102}$	12. $\frac{111}{222}$	18. $\frac{80}{200}$	24. $\frac{18}{50}$	30. $\frac{184}{208}$

40. Reduction to Like Denomination by Inspection. Changing fractions to fractions of like denomination is called *reducing to a common denominator.* Fractions of like denominators are *similar fractions.* The practice problems here given deal with the more commonly used fractions and are intended to develop the power of readily thinking one fraction in terms of another; *e.g.* $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{9}$, as $\frac{9}{18}$, $\frac{3}{18}$, and $\frac{2}{18}$. This is one of the operations in fractions most frequently needed.

DRILLS

1. $\frac{1}{2}$ = how many 4ths, 6ths, 10ths, 16ths, 18ths, 24ths, 28ths, 36ths, 72ds, 84ths, 60ths, 56ths, 42ds, 50ths, 44ths, 70ths?

2. $\frac{1}{3}$ = how many 9ths, 15ths, 21sts, 24ths, 27ths, 36ths, 45ths, 48ths, 51sts, 54ths, 63ds, 72ds?

3. $\frac{1}{4}$ = how many 8ths, 12ths, 16ths, 24ths, 28ths, 32ds, 36ths, 48ths, 52ds, 56ths, 60ths?

4. $\frac{1}{5}$ = how many 15ths, 25ths, 30ths, 35ths, 45ths, 50ths, 55ths, 65ths?

5. $\frac{1}{6}$ = how many 12ths, 18ths, 24ths, 30ths, 36ths, 42ds, 48ths, 54ths, 60ths, 66ths, 72ds?

6. $\frac{1}{8}$ = how many 16ths, 24ths, 32ds, 40ths, 48ths, 56ths, 64ths, 72ds?

7. $\frac{1}{7}$ = how many 14ths, 21sts, 28ths, 35ths, 42ds, 49ths, 56ths, 63ds, 70ths?

8. $\frac{1}{10}$ = how many 20ths, 30ths, 40ths, 50ths, 60ths, 70ths?

9. $\frac{1}{9}$ = how many 18ths, 27ths, 36ths, 45ths, 54ths, 63ds?

10. $\frac{1}{11}$ = how many 22ds, 33ds, 44ths, 55ths, 66ths, 77ths?

11. $\frac{1}{12}$ = how many 24ths, 36ths, 48ths, 60ths, 72ds?

12. $\frac{1}{13}$ = how many 26ths, 39ths, 52ds, 65ths, 78ths?

13. $\frac{1}{14}$ = how many 28ths, 42ds, 56ths, 70ths?

14. $\frac{1}{15}$ = how many 30ths, 45ths, 60ths, 75ths?

15. $\frac{1}{16}$ = how many 40ths, 60ths, 80ths?

16. From comparison with their equivalents, as required in the above problems, to what could one change or reduce halves, fourths, and eighths so that they would be of the same units or kinds of parts? What halves,

fourths, eighths, and sixths? What halves, fourths, eighths, sixths, and twelfths?

17. Reduce thirds, sixths, and ninths to common denominators. 3ds, 6ths, 12ths, and 15ths. 3ds, 6ths, 9ths, and 12ths.

41. Operations with Easily Reducible Fractions.

1. $\$5 + \$3 + \$2 = \$10$. 23 bu. + 15 bu. + 12 bu. = 50 bu.

   Likewise $\frac{3}{4} + \frac{5}{4} + \frac{6}{4} + \frac{9}{4} = 2\frac{3}{4}$, or $5\frac{3}{4}$.

2. $\$56 - \$12 = \$44$. 24 pk. − 19 pk. = 5 pk.

   Likewise $\frac{8}{9} - \frac{5}{9} = \frac{3}{9}$, or $\frac{1}{3}$.

3. $\$81 \div \$9 = 9$. 72 bu. ÷ 6 bu. = 12.

   Likewise $\frac{24}{30} \div \frac{3}{30} = 24 \div 3$, or 8.

Take note that in the *division of fractions having the same denominator, the denominators are disregarded, and the quotient of the fractions is the quotient of the numerators.*

REDUCE TO SIMILAR FRACTIONS BY INSPECTION AND ADD:

1. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}$
2. $\frac{3}{5}, \frac{2}{3}, \frac{3}{10}, \frac{5}{6}, \frac{2}{15}$
3. $\frac{7}{8}, \frac{3}{4}, \frac{1}{16}, \frac{1}{2}, \frac{5}{32}$
4. $\frac{3}{5}, \frac{6}{7}, \frac{5}{14}, \frac{3}{28}$
5. $\frac{2}{3}, \frac{5}{8}, \frac{7}{12}, \frac{3}{16}$
6. $\frac{1}{4}, \frac{4}{16}, \frac{5}{13}, \frac{5}{52}$
7. $\frac{7}{9}, \frac{2}{36}, \frac{3}{4}, \frac{15}{18}$
8. $\frac{1}{2}, \frac{5}{8}, \frac{2}{3}, \frac{4}{9}$

SUBTRACT:

1. $\frac{2}{14} - \frac{3}{7}$
2. $\frac{5}{8} - \frac{3}{7}$
3. $\frac{2}{3} - \frac{1}{4}$
4. $\frac{1}{2} - \frac{1}{16}$
5. $\frac{4}{5} - \frac{3}{4}$
6. $\frac{7}{12} - \frac{9}{16}$
7. $\frac{9}{13} - \frac{5}{52}$
8. $\frac{1}{9} - \frac{1}{27}$
9. $15\frac{1}{3} - 7\frac{1}{3}$
10. $16\frac{3}{4} - 5\frac{3}{8}$
11. $163 - 98\frac{4}{7}$
12. $85 - 14\frac{7}{8}$

DIVIDE:

NOTE. — All easily reducible fractions are more quickly and accurately divided by reducing to similar fractions. In practice, write the numerators only.

$\frac{3}{8} \div \frac{5}{12} = \frac{9}{24} \div \frac{10}{24} = \frac{9}{10}$

1. $\frac{2}{3} \div \frac{3}{4}$
2. $\frac{6}{7} \div \frac{1}{4}$
3. $\frac{8}{17} \div \frac{3}{34}$
4. $\frac{5}{8} \div \frac{1}{7}$
5. $\frac{30}{96} \div \frac{5}{24}$
6. $\frac{32}{56} \div \frac{4}{14}$
7. $\frac{5}{6} \div \frac{2}{3}$
8. $\frac{7}{11} \div \frac{5}{7}$
9. $16\frac{1}{4} \div 4\frac{1}{16}$
10. $\frac{6}{7} \div 4\frac{1}{4}$
11. $6\frac{3}{8} \div 1\frac{7}{24}$
12. $\frac{35}{108} \div \frac{7}{12}$

42. Addition and Subtraction through finding the Least Common Multiple. For fractions *not* easily reducible to a common denominator by inspection, the method of finding the Least Common Multiple of all the denominators is used in order to know the least denomination to which all of the fractions may be reduced.

PROBLEM. — Add $\frac{2}{3}$, $\frac{4}{9}$, $8\frac{5}{6}$, $\frac{5}{14}$, $5\frac{6}{7}$, $\frac{9}{14}$, $7\frac{1}{2}$, $\frac{11}{12}$.

In order to find the Least Common Multiple, it is usual to write all the denominators, then strike out any one that is repeated or that is a factor of any other; as, 3, 6, 14, 7, and 2. The remaining numbers are then divided by any divisor of two or more of them, bringing down any number not divisible. This is repeated, if necessary, until no two of the numbers remaining have a common divisor. The product of the divisors and all of the numbers remaining will be the Least Common Multiple of the given numbers.

$$\begin{array}{r|llll} 2) & \cancel{3} & \cancel{9} & \cancel{6} & \cancel{14} \quad \cancel{7} \quad 14 \quad \cancel{2} \quad 12 \\ \hline 3) & & 9 & & 7 \qquad\quad 6 \\ \hline & & 3 & & 7 \qquad\quad 2 \end{array}$$

$2 \times 3 \times 3 \times 7 \times 2 = 252$, L.C.M.

43. To reduce to a Common Denominator. Since 252 is a multiple of each of the denominators of the fractions given in the preceding section, each fraction may now be reduced to an equivalent fraction having 252 for its denominator. To do this, it is well to arrange the fractions vertically, placing the desired denominator below a line and directly beneath the fractions. To the right of a line drawn vertically, and opposite each fraction, is written the numerator of the equivalent fraction. The method used in finding this equivalent fraction is the same as by inspection. (Sec. 40.)

$\frac{2}{3}$	168
$\frac{4}{9}$	112
$8\frac{5}{6}$	210
$\frac{5}{14}$	90
$5\frac{6}{7}$	216
$\frac{9}{14}$	162
$7\frac{1}{2}$	126
$\frac{11}{12}$	231

$$\begin{array}{r|l} 252) & 1315 (5 \\ & \underline{1260} \\ & \frac{55}{252} \end{array}$$

$\frac{1}{3} = \frac{84}{252}$ (*i.e.* $\frac{1}{3}$ of $\frac{252}{252}$), and $\frac{2}{3} = \frac{168}{252}$. $\frac{1}{9} = \frac{28}{252}$ and $\frac{4}{9} = \frac{112}{252}$. $\frac{1}{7} = \frac{36}{252}$ and $\frac{6}{7} = \frac{216}{252}$, etc. Add-

$$\begin{array}{r} 20 \\ 5\frac{55}{252} \\ \hline 25\frac{55}{252} \end{array}$$

ing the numerators, the sum of the fractions is seen to be $\frac{165}{252}$ or 5 and $\frac{55}{252}$. Added to 20, the sum of the whole numbers, the sum required is found to be 25 and $\frac{55}{252}$.

PROBLEMS

1. Add 7, $8\frac{1}{4}$, $9\frac{7}{16}$, $6\frac{5}{8}$, $5\frac{9}{16}$, and $3\frac{1}{2}$.
2. Add $\frac{1}{2}$, $\frac{1}{6}$, $5\frac{3}{4}$, $12\frac{3}{11}$, and $\frac{8}{17}$.
3. Subtract $\frac{11}{18}$ from $7\frac{13}{21}$.
4. Subtract $\frac{29}{173}$ from $256\frac{3}{4}$.
5. From $1\frac{3}{7} + 11\frac{1}{7}$ take $12\frac{1}{2} - 9\frac{3}{5}$.
6. From the sum of $75\frac{1}{3}$ and $94\frac{1}{4}$ take the sum of $36\frac{5}{16}$ and $24\frac{3}{4}$.

44. Multiplication of Fractions and Whole Numbers.

(*a*) Five times 3 bu. = ? Five times 3 fourths = ? Five times $\frac{3}{4}$ = ? $\frac{3}{4} \times 5$ = ? How many wholes? (Use multiplication sign correctly. Sec. 13.)

$$5 \text{ times } \tfrac{3}{4} = \tfrac{15}{4} \text{ or } 3\tfrac{3}{4}.$$

(*b*) $5 \times \frac{3}{4}$ = ? This may also be written, $\frac{3}{4}$ of 5 = ? In the latter form it is recognized as finding the value of fractional parts. See Chapter IV.

$$\tfrac{3}{4} \text{ of } 5 = \tfrac{5}{4} \times 3 = \tfrac{15}{4} \text{ or } 3\tfrac{3}{4}.$$

A fraction may be multiplied *by multiplying its numerator. Dividing the denominator* also multiplies the fraction by increasing the size of the denomination. A fraction may be used as a multiplier *by dividing by its denominator and multiplying the quotient by the numerator.* This process may sometimes be shortened by first multiplying by the numerator and then dividing by the denominator.

In multiplying mixed numbers, the whole numbers and fractions should be multiplied separately and the partial products added. Solve each problem in the shortest possible way.

PROBLEMS

1. $348 \times \frac{3}{4} = ?$
2. $\frac{1}{17} \times 8 = ?$
3. $\frac{5}{7}$ of $721 = ?$
4. $12\frac{1}{4} \times 8 = ?$
5. $284 \times 3\frac{1}{8} = ?$
6. $71\frac{3}{4} \times 3 = ?$
7. $256 \times \frac{7}{8} = ?$
8. $5\frac{7}{11} \times 9.$

45. Multiplication of Fractions.

SOLVE:

1. $\frac{2}{3}$ of $\frac{9}{10}$.

Since $\frac{1}{3}$ of $\frac{9}{10}$ is $\frac{3}{10}$, $\frac{2}{3}$ is $\frac{6}{10}$ or $\frac{3}{5}$.

2. Find $\frac{3}{8}$ of $\frac{4}{5}$.

Since $\frac{1}{8}$ of $\frac{4}{5}$ is $\frac{4}{15}$, $\frac{3}{8}$ is $\frac{3}{15}$.

NOTE. — $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$, and $\frac{3}{8} \times \frac{4}{5} = \frac{3}{15}$; that is, the product of the numerators is divided by the product of the denominators.

3. Find $\frac{3}{5}$ of $\frac{5}{6}$.

Since $\frac{1}{5}$ of $\frac{5}{6}$ $(\frac{5}{6} \times \frac{1}{5}) = \frac{1}{6}$, $\frac{3}{5}$ of $\frac{5}{6} = \frac{3}{6}$ or $\frac{1}{2}$.

These steps may be performed together and shortened by writing the fractions, canceling the factors common to the numerators and denominators, and multiplying the remaining factors in the numerators for the numerator of the product, and the remaining factors in the denominators for the denominator of the product. Thus,

$$\frac{\cancel{3}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} = \frac{1}{2} \quad \text{and} \quad \frac{\overset{2}{\cancel{3}}}{\cancel{20}} \text{ of } \frac{\overset{6}{\cancel{24}}}{\cancel{52}} = \frac{2 \times 6}{5 \times 13} = \frac{12}{65}.$$

4. Multiply $5\frac{3}{8}$ by $6\frac{3}{4}$.

Mixed numbers should ordinarily be reduced to fractions, then multiplied as in 3. Thus,

$$5\frac{3}{8} \times 6\frac{3}{4} = \frac{17}{\cancel{8}} \times \frac{\overset{9}{\cancel{27}}}{4} = \frac{17 \times 9}{4} = \frac{153}{4} \text{ or } 38\frac{1}{4}.$$

5. Multiply $144\frac{2}{5}$ by $9\frac{3}{4}$.

Reducing a large number, like $144\frac{2}{5}$, would give too large a numerator to be handled readily. In such cases, the whole number and the fraction of the multiplicand may be multiplied separately by the whole number and the fraction of the multiplier, and the partial products added. Thus,

$$\begin{array}{rl} 144\frac{2}{5} & \\ 9\frac{3}{4} & \\ \hline \frac{3}{10} & = (\frac{2}{5} \times \frac{3}{4}) \\ 108 & = (\frac{3}{4} \times 144) \\ 3\frac{6}{10} & = (\frac{2}{5} \times 9) \\ 1296 & = (144 \times 9) \\ \hline 1407\frac{9}{10} & \end{array}$$

PROBLEMS

1. $\frac{3}{4} \times \frac{2}{3} \times \frac{4}{5} = ?$
2. $\frac{5}{8} \times \frac{3}{5} \times \frac{4}{7} = ?$
3. $\frac{3}{7} \times \frac{5}{11} \times \frac{7}{24} = ?$
4. $\frac{9}{10} \times \frac{2}{7} \times \frac{5}{9} \times \frac{1}{4} = ?$
5. $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{4}{5} = ?$
6. $2\frac{1}{4} \times 3\frac{1}{3} = ?$
7. $\frac{6}{7} \times 1\frac{3}{4} = ?$
8. $5\frac{1}{3} \times 7\frac{1}{7} = ?$
9. $\frac{8}{15}$ of $2\frac{1}{4} \times \frac{1}{3}$ of $7\frac{1}{3} = ?$
10. $9\frac{6}{7} \times 6\frac{3}{4} = ?$

11. A man owns $\frac{3}{5}$ of a store and sells $\frac{4}{5}$ of his share for $5000. What was the store worth, at this rate?

Find cost of:

12. 127 bu. wheat at $62\frac{1}{2}$¢ per bushel.
13. 125 cords of wood at $\$6\frac{1}{4}$ per cord.
14. $17\frac{3}{4}$ tons coal at $\$3\frac{3}{4}$ per ton.
15. 26 bu. clover seed at $\$7\frac{1}{2}$ per bushel.
16. $22\frac{1}{3}$ thousand feet lumber at $\$17\frac{1}{2}$ per M.

17. A merchant sold 25 yd. satin at $\$1\frac{1}{4}$ per yard, $26\frac{1}{2}$ yd. silk at $\$2\frac{1}{2}$, 26 yd. carpet at $\$\frac{7}{8}$, 56 yd. calico at $3\frac{3}{4}$¢. What was the amount of the sale?

18. Having bought $\frac{4}{5}$ of a farm of 180 acres, I sell $\frac{3}{4}$ of my share at $\$40$ per acre. How much do I receive for it?

19. A quantity of provisions will last 25 men $12\frac{3}{4}$ days. How long will it last one man?

46. Division of Fractions. Division of fractions which are not easily solved by the method in Sec. 41 is most readily performed by the method of inversion. In the latter, the terms of the divisor are inverted and the fractions are multiplied. Thus,

$$\tfrac{2}{3} \div \tfrac{5}{6} = \tfrac{2}{3} \times \tfrac{6}{5} = \tfrac{4}{5}. \quad \text{(Sec. 45.)}$$

As stated in Sec. 39, we cannot divide fractions without first reducing them to like denominations. In the division, then, of dissimilar fractions the two steps are: first, reduction to like denomination, and second, the division of the numerators. These two steps are accomplished simultaneously by the inversion of the terms of the divisor and multiplying, thereby shortening the written process. Thus, placing the denominator, 6, above the line, forms, with the denominator of the dividend, $\frac{6}{3}$ or 2. This is the number necessary by which to multiply the numerator and denominator of the fraction ($\frac{2}{3}$) to change it to sixths, that it may be of like units with the divisor ($\frac{5}{6}$). If this multiplication were actually made, the $\frac{2}{3}$ would now be $\frac{4}{6}$. The placing of the numerator of the dividend, 5, below the line indicates the division of 4 (the numerator of the dividend) by it, or performs the second step in the operation. This gives, as the result, $\frac{4}{5}$, the required result, since the numerators only are to be divided when the fractions are similar. (Sec. 41.) The ease with which inversion and simple multiplication accomplishes these steps as $\frac{2}{3} \times \frac{6}{5} = \frac{4}{5}$, explains the economy in its use for all problems not mentally reducible to a common denominator.

NOTE. — The example used in the above illustration could be more readily solved by mentally reducing both fractions to sixths, when the quotient would be readily seen to be $\frac{4}{5}$. The method of inversion is only shorter when the fractions have much larger terms, or are not readily reducible.

A fraction may be divided by a whole number by dividing the numera-

tor, or by multiplying the denominator. The latter divides by decreasing the size of the denominator. A whole number may be divided by a fraction by reducing it to a similar fraction and dividing the numerators.

DIVIDE:

1. $\frac{4}{15}$ by $\frac{1}{7}$. 3. $\frac{5}{14}$ by $\frac{1}{13}$. 5. $\frac{11}{13}$ by $\frac{1}{17}$. 7. $\frac{11}{12}$ by $\frac{1}{3}$. 9. $\frac{11}{12}$ by $\frac{1}{11}$.
2. $\frac{11}{17}$ by $\frac{1}{3}$. 4. $\frac{17}{18}$ by $\frac{1}{3}$. 6. $\frac{11}{12}$ by $\frac{19}{17}$. 8. $\frac{1}{3}$ by $\frac{1}{10}$. 10. $\frac{19}{3}$ by 16.
11. $\frac{11}{17}$ by $\frac{19}{11}$. 13. $\frac{21}{11}$ by 14. 15. 8 by $\frac{1}{3}$.
12. $\frac{1}{101}$ by $\frac{11}{11}$. 14. $\frac{1}{4}$ by 13. 16. 213 by $\frac{11}{11}$.

47. Decimal Fractions. By definition (Sec. 38), a decimal fraction is a common fraction whose denominator is 1 with ciphers annexed. Thus, $\frac{105}{1000}$, $\frac{17}{10000}$, and $\frac{2105}{10}$ are decimal fractions.

These decimal fractions may be written without the denominator (Sec. 18), by pointing off from the right of the numerator one less than the number of digits in the denominator. This would always be equal to the number of ciphers in the denominator. The above decimal fractions would thus be written as decimals: .105, .0017, and 210.5.

The position of the decimal point in a decimal always indicates, then, the number of ciphers at the right of the 1 in the denominator, if written as a decimal fraction. Decimals may thus be changed to common fractions by writing as decimal fractions, and then reducing, if desired, to lower terms.

1. Write as decimals: $\frac{15}{100}$, $\frac{1735}{100000}$, $\frac{55}{1000}$, $\frac{11375}{10000}$, and $\frac{1015}{1000}$.
2. Write as decimal fractions: 5.625, .00052, 1.1, and 32.0225.
3. Change to decimal fractions and reduce to lower terms: .125, .00875, .0625, 75.015, and 300.05

48. Changing Fractions to Decimals.

(*a*) When the denominator of the required fraction is known.

EXAMPLE. $\frac{3}{16}$ = how many thousandths?

$\frac{3}{16} = \frac{3}{16}$ of $\frac{1000}{1000} = \frac{3000}{16000}$ thousandths or .187$\frac{1}{2}$.

Ordinary business problems do not require the use of decimals beyond the thousandths. Practice in reduction from fractions to decimals should largely be, then, to acquire facility in changing to tenths, hundredths, and thousandths, and in writing at sight the decimal form of the simpler fractions. After the method of reduction is understood, these decimal equivalents should be drilled upon until they can be written at sight.

(*b*) When merely a change of form is desired.

EXAMPLE. Reduce $\frac{3}{16}$ to a decimal.

$$16\overline{)3.0000}$$
$$.1875$$

This is the method of reduction usually given. It consists of annexing decimal ciphers to the numerator, and dividing by the denominator. The business necessities for the use of this method are not frequent. It should, however, be thoroughly understood.

CHANGE TO DECIMALS:

To hundredths.		To thousandths.	
1. $\frac{9}{10}$	6. $\frac{3}{4}$	1. $\frac{9}{125}$	6. $\frac{15}{16}$
2. $\frac{3}{10}$	7. $\frac{2}{5}$	2. $\frac{3}{500}$	7. $\frac{1}{64}$
3. $\frac{10}{25}$	8. $\frac{6}{7}$	3. $\frac{43}{500}$	8. $\frac{11}{125}$
4. $\frac{44}{50}$	9. $\frac{4}{125}$	4. $\frac{15}{40}$	9. $\frac{19}{72}$
5. $\frac{73}{100}$	10. $\frac{9}{50}$	5. $\frac{24}{64}$	10. $\frac{55}{64}$

REDUCE TO DECIMALS:

Do not carry beyond five places.

1. $\frac{5}{11}$	6. $\frac{7}{8}$
2. $\frac{19}{1}$	7. $\frac{5}{12}$
3. $\frac{5}{14}$	8. $\frac{14}{15}$
4. $\frac{9}{700}$	9. $\frac{12}{15}$
5. $\frac{6}{13}$	10. $3\frac{1}{6}$

MISCELLANEOUS PROBLEMS IN FRACTIONS

1. An estate is divided among three sons so that the first gets $\frac{1}{4}$, the second $\frac{1}{10}$, and the third the remainder, $3600. What is the amount of the estate?

2. If a clerk spends $\frac{1}{2}$ of his weekly salary for board, $\frac{1}{4}$ for clothing, and $\frac{1}{8}$ for books and papers, and has left $4, what is his salary?

3. If I sell a house for $3600, thereby gaining $\frac{1}{5}$, what was the cost of the house?

4. A real estate agent rents a house for $850, which is $\frac{1}{7}$ of its cost. What is its cost?

5. A book dealer paid $21 for a set of books and sold them for $24. The gain was what part of the cost?

6. A farmer sold two horses for $48 each. On one he lost $\frac{1}{7}$ of the cost, on the other he gained $\frac{1}{7}$ of the cost. How much did he gain or lose by the transaction?

7. If 3 yd. of cloth cost 37½¢, how much change would you receive from a $5 bill if you buy 15 yd.?

8. A grain dealer had $14,000. He spent $\frac{2}{7}$ of it for wheat at 75¢ a bushel, and $\frac{2}{5}$ of the remainder for oats at 30¢ a bushel. How many bushels of each did he buy, and how much money had he left?

9. A man spent $\frac{1}{4}$ of his money for a suit, $\frac{1}{2}$ of the remainder for a shotgun, and has left $40. How much had he at first?

10. A locomotive runs $\frac{3}{4}$ of a mile in $\frac{5}{6}$ of a minute. At what rate per hour does it run?

11. The silver dollar weighs 412.5 grains; $\frac{1}{10}$ of its weight is alloy. How many grains of pure silver are there in one dollar?

12. Gold is 19.36 times as heavy as water; copper, 8.97; lead, 11.36. Find the weight of a cubic foot of each, if a cubic foot of water weighs 62½ lb.

13. The total crop of cotton in the United States in a certain year was 10,758,000 bales. Of this amount 6,482,849 bales were exported to Europe. What fraction (decimal) of the crop was exported? Retained at home?

14. If a coal dealer gains $\frac{1}{4}$ by selling coal for $8 a ton, how much would he gain on a sale of 8.8 tons?

15. After paying \$74.85 for mileage, \$37.50 for hotel bills, and \$13.65 for sundry expenses, a traveling agent finds that he has expended $\frac{2}{3}$ of his money. How much had he at first?

16. Find the value of a sheep which dressed as follows:

Leg	22.5 lb., $12\frac{1}{2}\not{c}$
Loin	17.5 lb., $9\frac{3}{4}\not{c}$
Rib	14.8 lb., $9\frac{1}{4}\not{c}$
Chuck	19.6 lb., $2\frac{1}{4}\not{c}$

17. The price of corn as quoted at the close of the market each day was as follows: Monday, $58\frac{1}{2}\not{c}$; Tuesday, $58\frac{3}{4}\not{c}$; Wednesday, $56\frac{1}{4}\not{c}$; Thursday, $57\frac{3}{8}\not{c}$; Friday, $57\frac{1}{8}\not{c}$; Saturday, $56\frac{1}{4}\not{c}$. Find the average price for the week.

18. A can do a piece of work in 4 days, B can do it in 5 days. How long will it take them both to do it?

19. A general store's sales of dry goods for a month amounted to \$6300, and $\frac{5}{7}$ of the sales of dry goods was $\frac{2}{3}$ the sales of groceries. What was the sales of groceries?

20. A bank teller received during the day \$60,000 in silver and paper money. There was $\frac{1}{5}$ as much silver as paper money. How much of each did he receive?

21. An automobile cost \$1600. If $\frac{1}{5}$ the cost of the automobile is 4 times the cost of a carriage, what is the cost of a carriage?

22. A grocer bought eggs at the rate of 4 for $5\not{c}$ and sold them at the rate of 5 for $9\not{c}$. How much did he gain on each dozen?

23. A piece of cloth is 20 yd. long and $\frac{3}{4}$ yd. wide. How wide is another piece which is 12 yd. long and contains as many square yards as the first?

24. If I pay \$48 for a buggy after receiving a discount of $\frac{1}{3}$, and a further discount of $\frac{1}{7}$ of the latter price for cash, what was the asking price?

25. A broker sold stocks at \$82 and gained $\frac{1}{10}$. What would he have gained or lost had he sold them a few days later when they were quoted at \$64?

26. If an ordinary gas burner consumes $\frac{1}{100}$ cu. ft. of gas per second, what would be the cost per hour to light a room with 50 burners at \$1.25 per thousand cubic feet? If a Welsbach burner consumes $\frac{1}{4}$ as much gas, how much would be saved in a day of 6 hours by installing Welsbach burners? How much in a month of 30 days?

27. A owned $\frac{4}{5}$ of a store and sold $\frac{3}{4}$ of his share to C. C sold $\frac{1}{2}$ of what he bought to B for $4000. At this rate, what was the store worth?

28. James and John hire a pasture for $35. James puts in 4 cows and John puts in 3. What must each pay?

29. A merchant sold a quantity of coffee for $1280, and thereby gained $\frac{1}{5}$ of the cost. If he had sold it for $1000, would he have gained or lost, and how much?

30. The net profits of a business for two years were $6400. The second year's profits were $\frac{1}{7}$ greater than the first year's. What were the profits each year?

31. I paid $22,500 for two farms. If $\frac{2}{3}$ of the cost of one is equal to $\frac{4}{5}$ of the cost of the other, what did I pay for each one?

32. A merchant bought 300 crates of peaches at $87\frac{1}{2}¢$ a crate. He sold $\frac{1}{3}$ of them at an advance of $10¢$ a crate, $\frac{1}{2}$ of the remainder at $80¢$ a crate, and the remainder at a loss of $3\frac{1}{2}¢$ a crate. What did the merchant gain or lose?

33. A man invested $\frac{1}{3}$ of his money in bonds, $\frac{1}{4}$ of it in real estate, $\frac{1}{5}$ in mining stock, and the balance, $3900, in bank stock. How much did he have in all? How much in each investment?

34. A man spent $\frac{1}{7}$ of his money for a house, invested $\frac{1}{3}$ of the remainder in stocks, and had $3200 left. How much had he at first?

35. A tree fell, breaking in three pieces. The first was $\frac{2}{3}$ as long as the second, and the third was $\frac{1}{2}$ as long as the other two pieces. What was the length of each piece, if the total length was 180 ft.?

36. A speculator invested $\frac{1}{3}$ of his money and $600 in land, $\frac{1}{5}$ of his money and $250 in bank stock, $\frac{1}{8}$ of his money and $144 in bonds, and the remainder, which was $1400, in a house and lot. What did he invest in each kind of property?

37. George's money is $\frac{3}{4}$ of James'. James' money is $\frac{5}{6}$ of Clara's. Clara's is $1\frac{1}{2}$ times Daniel's. How much had each, if $\frac{1}{3}$ of George's money is $60?

VI

MEASURES OF LENGTH

49. Measurement. Measurement of quantity enters so largely into life that to make arithmetic really practical, our concepts of the units of measurement should be very accurate.

The first efforts of the learner should be toward the building of accurate concepts of the various units, rather than toward proficiency in repeating tables or in changing to higher or lower units.

NOTE. — Pupils should have the actual measurement units present to their senses. Do not make the mistake of talking of rods, miles, acres, etc., without bringing these quantities actually before you. Use a tape or string a rod long; view and walk a mile, or view and walk around an acre, etc. Practice judging length, extent, or weight, and test your accuracy by measurement.

50. The Unit of Length. The unit for measuring length is the *yard*. Formerly the unit for the United States was the same as the English yard, but the law of 1893 made the yard $\frac{3600}{3937}$ of the international unit, the meter. A standard yard is kept in the Bureau of Standards at Washington (Sec. 127).

TABLE

There are: 12 inches ($''$) in 1 foot ($'$).
3 feet in 1 *yard* ($^\circ$).
$5\frac{1}{2}$ yards in 1 rod.
320 rods in 1 mile.
1 mile = 5280 feet or 63,360 inches.

51. Surveyor's Measure. The unit of land measure has long been the Gunter's chain, 4 rods, or 66 feet long. This was divided into 100 parts or links, each link being 7.92 inches long. Eighty of these chains make a mile.

In civil engineering and at the customhouse, the inch and foot are divided into tenths, hundredths, and thousandths, in lieu of the usual subdivisions of halves, quarters, eighths, etc. This is indicative of a tendency in all measurements toward a larger use of decimal divisions. Some of the advantages of a decimal system of measurement, the strongest argument for the metric system, are thereby secured.

52. Nautical Units. Numerous special units have become established by usage in particular vocations. Many of these are of local use, others have varying values in different localities, and still others have become or are becoming obsolete. A few nautical units of length or distance are here given.

There are: 6 feet in 1 fathom.
1.15 statute miles in 1 geographic (sometimes called nautical) mile or knot.
3 geographic miles in 1 league.
60 geographic miles in 1 degree.
69.16 statute miles in 1 degree of the earth's equator.
360 degrees in 1 circumference of the earth.

CLASS EXERCISES

1. Mark off 1 foot in length on the board. Scan it carefully.

2. Mark off 1 foot without using the ruler. Test accuracy.

3. Draw a horizontal line 1 foot long; a vertical line; an oblique line. Test them.

4. Estimate the number of feet in length of your desk top; its width; the length and width of the door; of the window. Verify your estimates by measuring.

5. Estimate the height of the room; its width; its length. Test.

6. Practice judging length of articles in the room, and of distances, until you can estimate a foot with considerable accuracy. An inch. A yard.

7. Carefully measure off a rod. Estimate length in rods until you can do so accurately.

8. Make a chain, either Gunter's or 100-foot chain. In company with another pupil, if it can be arranged after school hours, measure off a mile. Mark its limit by some signal, so that the eye may judge the distance. Walk the mile. Estimate distances in miles and, if possible, verify estimates.

PROBLEMS

1. How many inches in 7 ft.? In 3 yd.? In a rod?

2. How many feet in 90 in.? In 4 rd.? In a half mile?

3. A ship travels 18 knots per hour. How many miles does she travel in 6 hours?

4. A garden is 90 ft. square. How many yards of fence will it take to inclose it?

5. A well is 25 yd. deep. What will be the cost of a pump stock, that reaches to the bottom, at 8¢ per foot?

6. If potato rows are 3 ft. apart, how many rows are there in a lot 4 rd. wide?

7. A gardener has a bed 16 ft. long and 6 ft. wide. He wishes to have 4 rows of plants, 6 in. apart, in the row. How many plants will it take? How far apart are the rows?

8. How many miles does a boy ride in a month of 26 days, if he rides 342 rd. to and from work each day?

9. If a man digs a ditch 3 rd. long in a day, how long will it take him at the same rate to dig a ditch ¼ mi. long?

10. If railroad ties are laid 18 in. apart (from center to center), how many ties will it take to lay a mile of track? What will they cost at 75¢ apiece?

11. What will it cost to place a hedge around a lot, the distance around which is 72 rd., if the plants are placed 6 in. apart and cost $4.25 per hundred?

12. The distance between two cities is 12 mi. What will the wire to build a telephone line cost at \$2.90 per hundredweight, if it weighs 1¼ lb. to the rod?

13. A barn roof is 84 ft. long. What will it cost to place an eave-trough along two sides, if the trough costs 55¢ per 10 ft. length?

14. The length of a rectangular lot is 100 yd., its width 40 yd. What will it cost to fence it with wire netting at \$3.60 per 150 ft., and posts, set 10 ft. apart, at 35¢ each?

15. A water company wishes to lay a line of pipe along a mile of street. If the pipe is worth \$1.25 per foot, what will it cost?

16. A gentleman has a field, the perimeter of which (*i.e.* the distance around it) is 320 rd. He wishes to build a fence of eight wires, with posts set 8 ft. apart around it. If the wire weighs 1 lb. to the rod and costs \$2.50 per hundredweight, and the posts cost 17¢ each, what will it cost for material to build the fence?

17. What will it cost to fence a lot, with boards 16 ft. long and 4 in. wide, at 10¢ per board, if the perimeter of the lot is 63 rd. and the fence is to be 6 boards high?

18. My lot is 32 rd. long and 20 rd. wide. I built a picket fence around it, using pickets 4 in. wide, and placing them 2 in. apart. How many pickets were required? What did they cost at \$3.25 per M?

19. A plot of ground is 4 rd. long and 2 rd. wide. How many strawberry plants will it take to set the plot in rows 2 ft. apart, running lengthwise, if the plants are set 10 in. apart in the row?

20. What will the material to build 125 mi. of railroad cost, if the rails are 30 ft. long, weigh 25 lb. to the foot, and cost \$28 per ton of 2000 lb., and the ties, costing 75¢ apiece, are laid 3050 to the mile?

21. What will it cost to build an electric line between two cities, 15 mi. apart; the rails weighing 25 lb. to the foot, and costing \$18 per ton; the ties costing 70¢ apiece, laid 2 ft. apart; the wire (one strand) weighing 5 lb. to the rod and costing \$10 per hundredweight; the posts being set 19 rd. apart, and costing 80¢ apiece?

22. A tennis court has four lines 78 ft. long, two lines 36 ft. long, two lines 27 ft. long, and one line 31 ft. long. What will the tape to mark these lines cost at 4¢ per foot?

23. How many yards of carpet will be required for a flight of 15 steps 1 ft. wide and 6 in. high, and a landing 6 ft. 8 in. wide? What will it cost at 70¢ per yard?

24. The two aisles of a church are each 6 ft. wide and 85 ft. long. At $1.85 per linear yard, what will the carpet cost to cover them?

25. In surveying the route for a proposed railroad the surveyors applied the Gunter's chain 6450 times. How many miles of road? How many rails, 30 ft. long, would it require? How many ties, counting 3050 to the mile?

26. A boat is rowed at the rate of 10 knots an hour. The current runs at the rate of 4 knots an hour. If the boat is rowed with the current, how many miles will it go in 10 hr.?

27. The elevator in the Washington monument goes 500 ft. above the base. How many rods of cable in the eight strands extending from bottom to top?

there are four square inches along one edge or in one row, and two such rows. There are, therefore, 2 times 4 square inches or 8 square inches in the figure.

In all rectangles there are as many square units in one row or along one edge as linear units along that edge. Likewise, there are as many rows of square units as linear units along the other edge. To find the area, then, *the number of square units in one row should be multiplied by the number of rows.*

In a room 10 by 12 feet, there are 10 rows of 12 square feet or 120 square feet in the floor. The multiplicand is thus to be always a number of square units, and the product, therefore, square units.

PROBLEMS

Find the area of the following rectangles:

1. 6 by 9 inches.
9 sq. in. × 6 = 54 sq. in.

2. 13 by 24 rods.

3. 16 by 87 yards.

4. 41 by 65 feet.

5. 11 by 47 inches.

6. 35 by 46 feet.

7. 15 by 36 rods.

8. 45 by 86 yards.

9. 32 by 54 inches.

10. 40 by 60 yards.

11. 24 by 48 feet.

12. A barn floor is 40 by 60 ft. What will it cost to cement it at 12¢ per square foot?

13. How many rolls of weather paper each containing 100 sq. ft. will it take to cover a roof 42 by 50 ft.?

14. A board fence 7 ft. high surrounds a lot 32 by 120 ft. How many square feet of boards in this fence? What would it cost to paint it at 4¢ a square foot?

15. A box is 6 by 8 in. and 4 in. high. How many square inches in its 4 sides?

NOTE. — Think of the sides as a rectangle having 4 rows of 28 sq. in.

16. A room is 12 by 18 ft. and 10 ft. high. How many square feet in the four walls? In the floor and ceiling?

17. What will it cost to tint the four walls of a room 15 by 18 ft. and 10 ft. high, at 40¢ a square yard? How many square yards in the ceiling? What will be the cost of tinting the ceiling at the same rate? What was the total cost of plastering the room at 25¢ per square yard?

18. What will it cost to build a walk of stone, 6 by 360 ft., at 60¢ per square foot?

19. A barn is 40 by 80 ft. and 25 ft. high. What will it cost to paint the four sides at 25¢ a square yard?

20. A hall floor is 15 by 30 ft. How many tiles 6 by 6 in. will it take to lay the floor?

21. There are 48 sq. ft. of boards on the side of a building. If each board is 6 in. wide, and there are 12 of them, how long are they?

22. What will it cost to calcimine the walls and ceiling of a room 18 by 15 ft., and 9 ft. high, at 20¢ per square yard?

23. A schoolroom is 30 by 40 ft. What will it cost to put a slate board along one side and end, the slate being 4 ft. wide and costing 18¢ per square foot?

24. How many bricks 4 by 8 in. will be needed for a walk 36 yd. long and 4 yd. wide? What will they cost at $6.50 per M?

25. How many square feet of sidewalk 6 ft. wide will be required to surround a lot 250 by 360 ft.? What will it cost to lay it at 12¢ a square foot?

26. The owner of a lot of ground 1080 ft. long and 600 ft. wide cuts two streets, each 75 ft. wide, through the middle, one running east and west, and the other running north and south. How many acres has he left? What will brick 6 by 9 in. cost to pave these streets at $7 per M?

27. A farmer owned a rectangular piece of land 40 by 80 rd. He sold four lots, 7 by 10 rd., 8 by 25 rd., 10 by 15 rd., and 15 by 20 rd. How many acres did he sell? How many remained? How many rods of fence will it take to fence the remaining land, if the lots are taken, one from each corner?

55. Squaring a Number. A rectangle which contains the same number of rows of square units as there are square units in each row is called a *square*. *The number of units in each row of a square is called the square root.*

CLASS EXERCISES

1. The square on 3 ft. has how many rows and how many square units in each row? How many square units in the square, or its area?

2. A square on 9 in. has how many square inches in each row?

3. When a square contains 25 sq. in., it is how many inches long and wide?

4. What is the square root of 9 sq. ft.? Of 16 sq. ft.? Of 25 sq. ft.?

5. What is the square root of: 81, 49, 100, 225, 2500, 625, 169?

6. Drill upon the following table:

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$20^2 = 400$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$25^2 = 625$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$50^2 = 2500$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$100^2 = 10000$

FINDING THE SQUARE ROOT

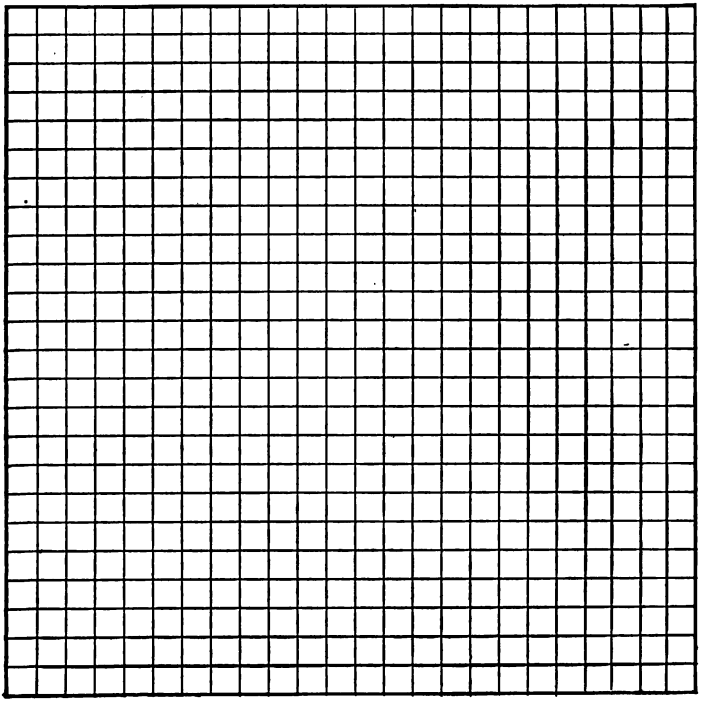
56. Root Periods in the Square. In squaring numbers from 1 to 10, it may be noted that the square of a number composed of one figure never contains less than one or more than two figures. Squaring 10 and 99, it will be seen that the square of a number composed of two figures has not less than three nor more than four figures. In the same way it may be seen that the square of a number composed of three figures contains not less than five nor more than six figures. It may be concluded, then, that for every two figures in a square there is one figure in the root. If we mark off the number whose square root we are seeking, into periods of two figures each, beginning at the right, we will get one figure in the root for each period.

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$99^2 = 9801$

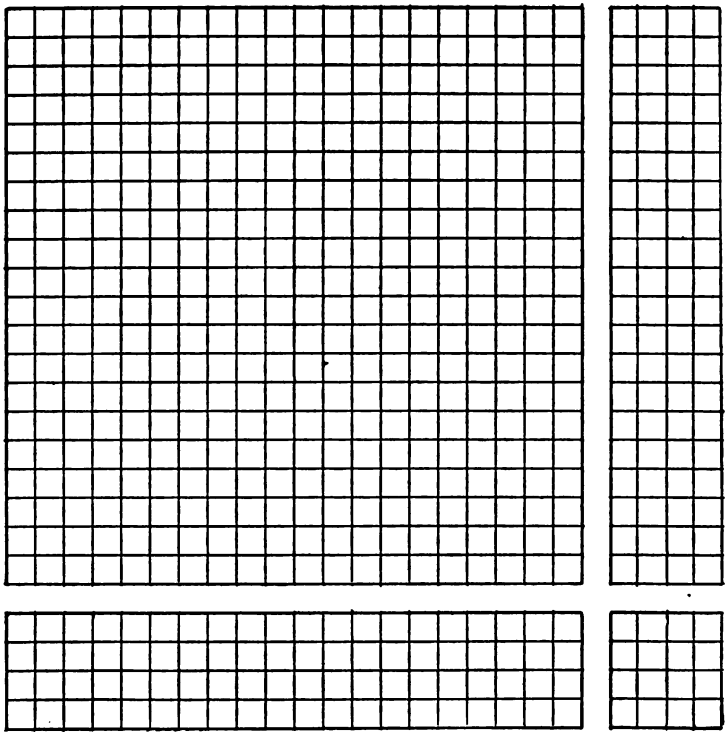
57. Square of a Number composed of Two Figures. Take a paper 24 in. square, and carefully mark it off into square inches. From a lower corner cut out the square on 20 in.,

and from the opposite upper corner cut out the square on 4 in. What are the dimensions of the two oblongs remaining after the squares have been removed? Observe that the square on 20 in. is the square of the tens of the number squared (24), and that the other square is the square of the units (4). The dimensions of the two oblongs are also, respectively, the same as the tens and units of the original number.

The area of the square of a number composed of two figures is, then, equal to the square of its tens plus two rectangles with the tens and units as dimensions, plus the square of its units.



A Square



THE SAME SQUARE SEPARATED

PROBLEM

58. Finding the Square Root. Find the side of a square whose area is 576 sq. ft., or extract the square root of 576.

	5′76 sq. ft. (24
	4 00
The square on 2 tens, or	1 76 sq. ft., the area of 2
20 sq. ft. × 2 = 40 sq. ft.	rectangles and smaller
4 sq. ft.	square.
44 sq. ft.	1 76 sq. ft. the square feet
Sq. ft. in one row of 2 rectangles and smaller square.	in 4 rows.

Marking off 576 according to Sec. 56, we find that its root will contain two figures, tens and units. From Sec. 57, also, we see that the second period, or 6 hundreds, contains the square of the second or tens figure.

Taking out the square of the largest tens (2 tens) contained in 6 hundreds, or 4 hundreds, we have left 176 sq. ft. as the area of the remaining two rectangles and the smaller square. We know (Sec. 57) that the rectangles have each two tens, or 20 sq. ft., along one edge or in one row. We do not as yet know the dimensions of the smaller square, but we know they are the same as the other dimension of the rectangles.

If the area of the rectangles is nearly 176 sq. ft., and the number of square feet along one edge of both is 40; then there must be nearly as many rows of 40 sq. ft. as 40 sq. ft. is contained in 176 sq. ft., or 4 rows. If this is correct, the smaller square is 4 ft. square, and along one edge there would be 4 sq. ft.

Along one edge of both the rectangles and the smaller square, there would be, then, 44 sq. ft., and if 4 rows wide, the area of the rectangles and square would be 176 sq. ft., or the same as the area remaining after removing the larger square.

The other dimension of the rectangles, therefore, must be 4 ft., and the units figure 4, making the square root 24.

STATEMENT OF PROCESS

The principles applied in finding the square root of larger numbers are the same as when the root contains two figures. The two periods to the left are treated as containing the square of the number composed of two figures, and when these are found, they are considered as the known tens' figure, and the units' figure is sought in the next period to the right, and so on, until the complete root has been found.

The following is a summary of the steps in the method:

(1) Beginning at the right, mark off periods of two figures each.

(2) By inspection use the square root of the largest square contained in the left-hand period as the first figure of the root, and subtract its square from that period, bringing down the next period to the right.

(3) Treating the first figure thus obtained as the tens and adding a cipher, we have the number of squares along one edge of one rectangle. Multiplying this by 2 and dividing the product into the number brought down (allowing some for the area of the smaller square), the probable width of the rectangles or second root is obtained.

(4) Add the root to the number of squares along one edge of both rectangles, and multiply the sum by the same figure (as indicating the number of rows) to complete the square.

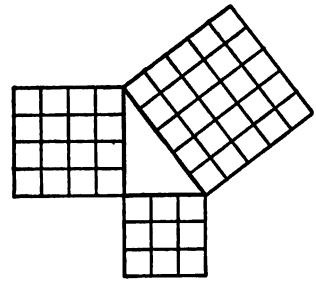
(5) Subtract this from the number brought down, and bring down the next period, proceeding as in (3) and (4).

59. Square Root of Decimals and Fractions. When the square consists of a decimal or a whole number and decimal, mark off periods right and left from the decimal point, and proceed as in whole numbers. The root figure for the first decimal period would be tenths; the second, hundredths, etc.

The square root of a fraction may be found by extracting the square root of the numerator for a new numerator and of the denominator for a new denominator.

60. Applications of Square Root to the Right Triangle. Draw a right triangle (Sec. 68) having a base of 3 inches and a perpendicular of 4 inches. What is the length of the hypotenuse?

Erect a square on the perpendicular. Erect a square on the hypotenuse, and on the base. How many square inches in the squares on the base and on the perpendicular? In the square on the hypotenuse?



Draw a right triangle with base and perpendicular, respectively, 6 and 8 inches. Draw squares on the three sides. Compare the area of the square on the hypotenuse of the triangle with those on the other two sides. These illustrate the geometrical truth, that *the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.*

PROBLEMS

FIND THE SQUARE ROOT OF:

1. 4225	**4.** 53,361	**7.** 72,984	**10.** $\frac{256}{4225}$
2. 5625	**5.** 17,424	**8.** 1900.96	**11.** $\frac{1024}{7225}$
3. 1225	**6.** 97,344	**9.** 97.8121	**12.** $\frac{576}{5625}$

13. The base of a triangle is 8 ft., the altitude 6 ft. What is the hypotenuse?

14. Find the base of a triangle whose altitude is 12 ft., hypotenuse 16 ft.

15. The hypotenuse of a triangle is 45 ft., the base 27 ft. Find the altitude.

16. A square farm contains 160 A. What is the length of one side?

17. A ladder is 25 ft. long and, when the foot is placed 15 ft. from the foot of a wall, just reaches the top. How high is the wall?

18. A lot is in the form of a right-angled triangle, whose base is 16 chains and altitude 12 chains. How many rods of fence will be needed to inclose it?

19. Find in rods the diagonal of a square field that contains 12 acres.

20. What is the distance from one lower corner to the opposite upper corner of a room, that is 36 by 72 ft. and 12 ft. high?

PRACTICAL APPLICATIONS OF AREA

61. Pitch of Roofs and Roofing. The degree of slant given to the sides of a roof is called its *pitch.*

When the height of the gable is one fourth the width of the building, the roof is said to have *one-fourth pitch.* When the height is one half the width, the roof has *one-half pitch;* when five eighths, it has a *five-eighths* or *Gothic pitch,* etc. One half is the pitch commonly used.

In estimating the cost of roofing in accordance with plans of a building under consideration, it is sometimes necessary, from the size of the building and the roof pitch, to find the dimensions of the two sides of a roof.

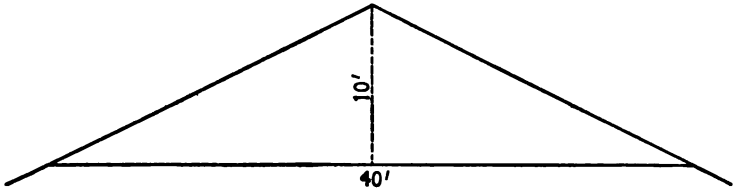
Knowing the width of the building, the degree of pitch will give the height of the gable or the perpendicular of the right triangle, one half the width of building forming the base. Applying the principle of Sec. 60, the length of the hypotenuse or the width of one side of the roof is then found. The distance the eaves extend over the sides of the building should be added to the roof width, and the extension of the gables over the ends should be added to the length of the building for the roof length.

The standard width of a shingle is 4 inches. This may vary, however, according to grade or style. In ordinary shingles there is no uniformity in width, but the average width in a bundle is supposed to be 4 inches.

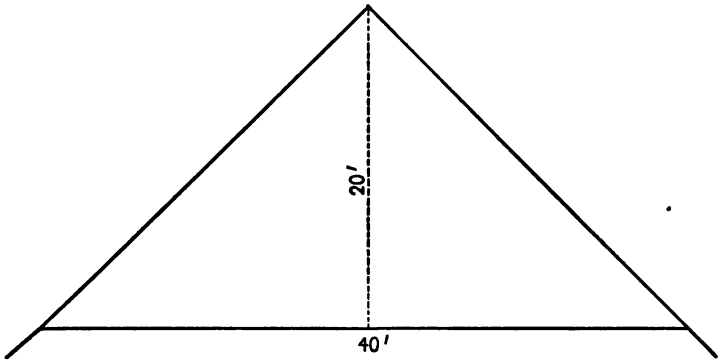
The standard exposure to the weather is, likewise, 4 inches. That is, in laying, all but 4 inches of the length of a shingle is covered. Thus, the average area of the exposed surface of a shingle is 16 sq. in.

In selecting the proper widths of shingles so that all joints

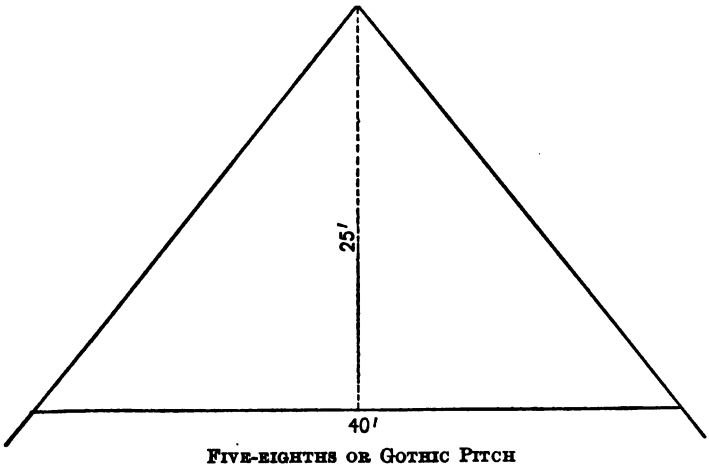
PITCH OF ROOFS



One-fourth Pitch



One-half Pitch



Five-eighths or Gothic Pitch

will be covered, and discarding defective shingles, there is more or less waste, varying with the grade of shingle used. *One ninth* is the amount usually allowed for such waste.

Thus, 9 shingles, at 16 sq. in. each, would cover a surface of 144 sq. in. or 1 sq. ft. With $\frac{1}{9}$ allowance of waste, 10 shingles would be required for 1 sq. ft., and 1000 shingles for 100 sq. ft., the surface unit. It is called the *square* (100 sq. ft.), and is used in roofing, flooring, slating, etc. As bundles are usually made up to contain 250 shingles, it would take *four to cover a square*, or one bundle would cover a surface of 25 sq. ft. Thus, a roof 24′ by 30′ would be 720 sq. ft. on each side, or 1440 sq. ft. on both sides. This would make 14.40 squares and would require $14\frac{1}{2}$ thousand shingles or 58 bundles.

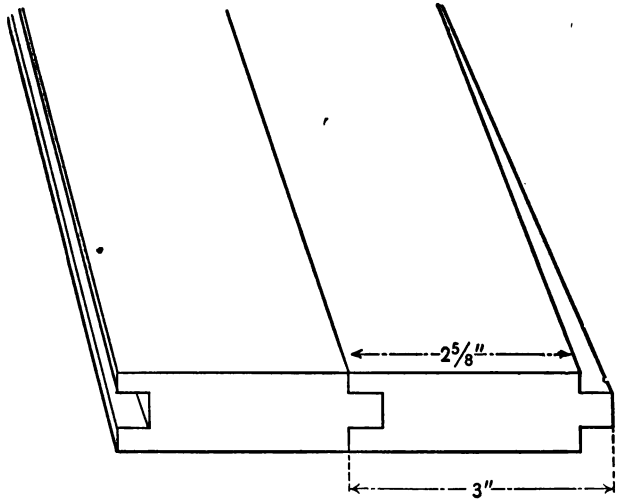
The outer edge of the roof is usually laid double. This is offset, however, by the width of the ridge board.

Bundles of shingles are not usually broken, so if the surface shows that a fraction of a bundle is needed, a whole bundle must be purchased.

When, for any purpose, an exposure other than the standard 4-inch is allowed, the number of shingles or bundles are found in the usual way, and the result is modified to meet the conditions. Thus, if the exposure is to be 3″ instead of 4″, $\frac{1}{4}$ more shingles will be required; if 2″, twice as many; if 5″, $\frac{1}{4}$ less; and if 6″, $\frac{1}{2}$ less. If the width of the shingle varies from the standard 4-inch, a correction may be made in the same way.

Roofing, other than shingling, is also figured by the square. In slating, the size of the slates varies from 6″ × 12″ to 16″ × 24″. The number of slates needed per square would, therefore, vary from 533 to 86. Contractors usually figure from prepared tables, showing the number of slate at a given size per square.

62. Flooring in Wood. Boards for flooring are tongued and grooved. This entails a loss in width of $\frac{3}{8}$ of an inch for each board. That is, a board bought as a 3-inch board will cover but $2\frac{5}{8}$ inches in width. Three-eighth inch loss on 3 inches is $\frac{1}{8}$-inch loss on 1 inch. In other words, each 1 inch of board width purchased will cover but $\frac{7}{8}$ inches of floor space. It will require $\frac{1}{7}$ more lumber, therefore, than it would if there were no waste.



TONGUED AND GROOVED

How much flooring would be required for a room $24' \times 32'$ if $3''$ flooring were used?

24 sq. ft. $\times$ 32 = 768 sq. ft. Surface area.

768 sq. ft. + $\frac{1}{7}$ of itself = 878 sq. ft. Flooring required.

Spruce or pine flooring is made 3 in., 4 in., or sometimes $5\frac{1}{2}$ in. wide. Hardwood flooring is but 2 in. or $2\frac{1}{2}$ in. wide, or even less. In the same way, allowances for loss from grooving, for the usual widths of flooring, is as follows: for 2 in. flooring, add $\frac{3}{13}$; for $2\frac{1}{2}$ in., add $\frac{3}{17}$; for 4 in., add $\frac{3}{29}$;

and for $5\frac{1}{2}$ in., add $\frac{5}{41}$ of the actual floor space. Prove the correctness of these fractions, as above. Carpenters charge by the *square* for laying floors.

PROBLEMS

1. The roof of a shed is 12 by 20 feet. How many slates 6 by 12 inches will be required to cover it? How many shingles? How many bundles of shingles?

2. What will it cost to shingle the two sides of a roof, if each side is 20 by 80 feet, with shingles costing $3.20 per M?

3. A barn is 40 by 80 feet, and the roof is $\frac{1}{2}$ pitch. How many shingles must I buy to roof it, if the roof extends 18 inches over each side and end?

4. The roof of a church is Gothic pitch. The building is 40 by 72 feet, and the roof extends 2 feet over each side and end. If shingles are laid 3 inches to the weather, what will be their cost at $4.50 per M?

5. The roof of a house is 56 feet long and each side is 18 feet wide. There are two verandas, each having a roof 10 by 20 feet. What will the shingles cost at $4.75 per M, if they are laid 5 inches to the weather?

6. A mill is 120 feet long. If each side of the roof is 32 feet wide, and it extends $2\frac{1}{2}$ feet over each side and end, what will the shingles cost at $3.80 per M, if they are laid 4 inches to the weather?

7. Figure the flooring lumber bill for house shown on pages 90–91, as follows: Hard pine at $48 per M on veranda, living room, dining room, and entire upper floor, except bathroom, which is to be tiled; two-inch maple at $72 on kitchen; two-inch oak at $80 on reception hall, library, and parlor; four-inch soft pine on porch, at $60; spruce, $5\frac{1}{2}$ inches, at $34 on laundry and cellar in basement.

63. Carpeting. The standard width of Kidderminster and Ingrain carpets is 36 inches. That of Brussels, Wiltons, Axminsters, etc., is 27 inches. Borders are usually $22\frac{1}{2}$ inches wide.

Rugs, whether pattern rugs or made up, are sold by the piece. All other carpeting is sold from the roll and by linear measure.

The direction the carpet is to be laid will often determine the cost, as there might be more waste if laid one way than if laid the other.

The length of each strip would be the length of the room plus the allowance for matching. The number of strips would be determined by dividing the width of the room by the width of the carpet, counting a full strip for all fractions, as any extra width must be cut off or turned under. Thus, for a room 15 by 24 feet, the strips would be 8 yards long, and with an allowance of 9 inches for matching, $8\frac{1}{4}$ yards long. If the carpet were Brussels, there would be 6 strips and 18 inches additional, requiring 7 strips. 7 strips of $8\frac{1}{4}$ yards each would be $57\frac{3}{4}$ yards.

PROBLEMS

1. How many yards of carpet 1 yard wide will it take to cover a floor 15 by 18 feet?

2. A room is 18 by 20 feet. How many yards of carpet $\frac{3}{4}$ yard wide will it take to cover the floor?

3. How many yards of carpet 27 inches wide will be required to cover the dining room in the floor plan on p. 90, if the strips run lengthwise? What will it cost at $1.20 per yard?

4. If the library shown on p. 90 is carpeted in the most economical way, with carpet 30 inches wide, what will be the cost at $1.25 per yard?

5. What will it cost to cover the kitchen of the house shown on p. 90 with linoleum costing $1.35 per square yard?

6. The three bedrooms in the plan of the second floor on page 91 are to be covered with carpet 1 yard wide, 8 inches waste in matching. What will be the cost at 95¢ per yard, if the strips run in the direction to leave the least waste?

7. The stairway in the floor plan (pp. 90–91) is 11 feet 4 inches high, the tread of each stair is 12 inches, and the riser 8 inches. What will be the cost of carpet at $1.35 per yard?

8. A rug $8' \times 13'$ is placed in the living room at a cost of $42.50. The remaining space is painted at a cost of 36¢ a square yard. What is the total cost for covering the floor?

64. Lathing and Plastering. The square yard is the unit by which estimates are made for the cost of lathing and plastering. Contracts are made at a given price per square yard of plastering, or lathing, or both.

In estimating the cost of lathing and plastering for scratch (first) and brown (second) coat on wood lath, the following quantities are generally allowed for 100 square yards: 1400 to 1500 laths (laths are put up in bundles of 50, and sold by the bundle or thousand); 10 pounds of 3-penny lathing nails; $2\frac{1}{2}$ barrels, or 500 pounds, of lime; 45 cubic feet, $1\frac{3}{4}$ loads, or 15 barrels, of sand; and 4 bushels of hair (hair is put up in bushel bundles, containing 5 packages). For the best quality of white coat, the estimate is 90 pounds of lime to 50 pounds of plaster of Paris, and 50 pounds of marble dust. Stucco is put up in bags containing 100 pounds each, and is sold by the ton; 900 to 1000 pounds are required for 100 square yards of surface.

Custom varies as to an allowance for doors and windows. Some contractors make no allowance, holding that the extra care and time needed in lathing and plastering around an opening make up for saving of material. When an allowance is made, it is usually 20 sq. ft. for each opening, and not the exact measurement. In estimating material alone, allowance is usually made for all openings. A fraction of a square yard is counted as a square yard in the final result.

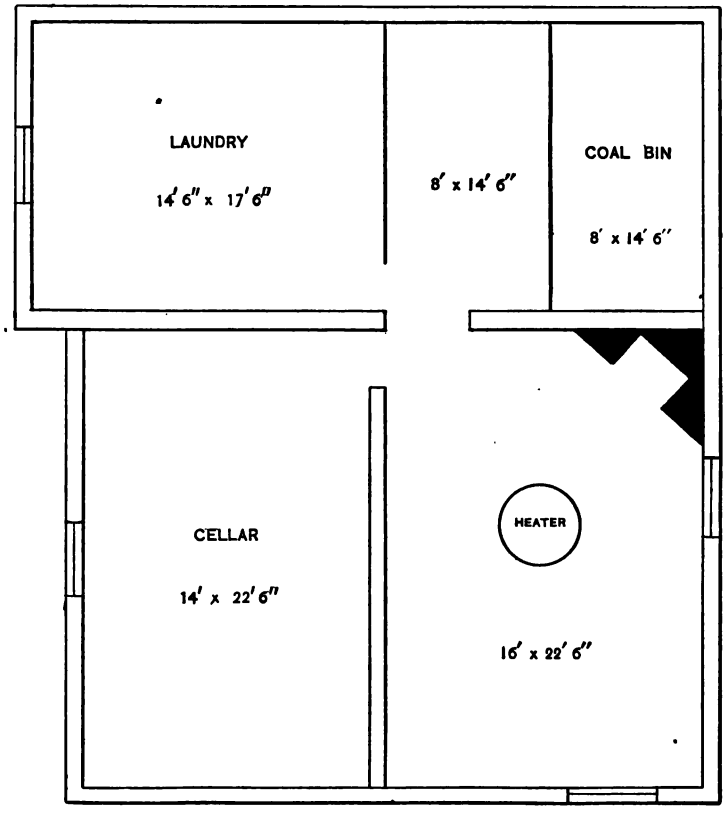
PROBLEMS

1. What will it cost to plaster a room 20 × 24 ft. and 10 ft. high, at 20¢ per square yard? How much lime, hair, and sand will it take for two coats?

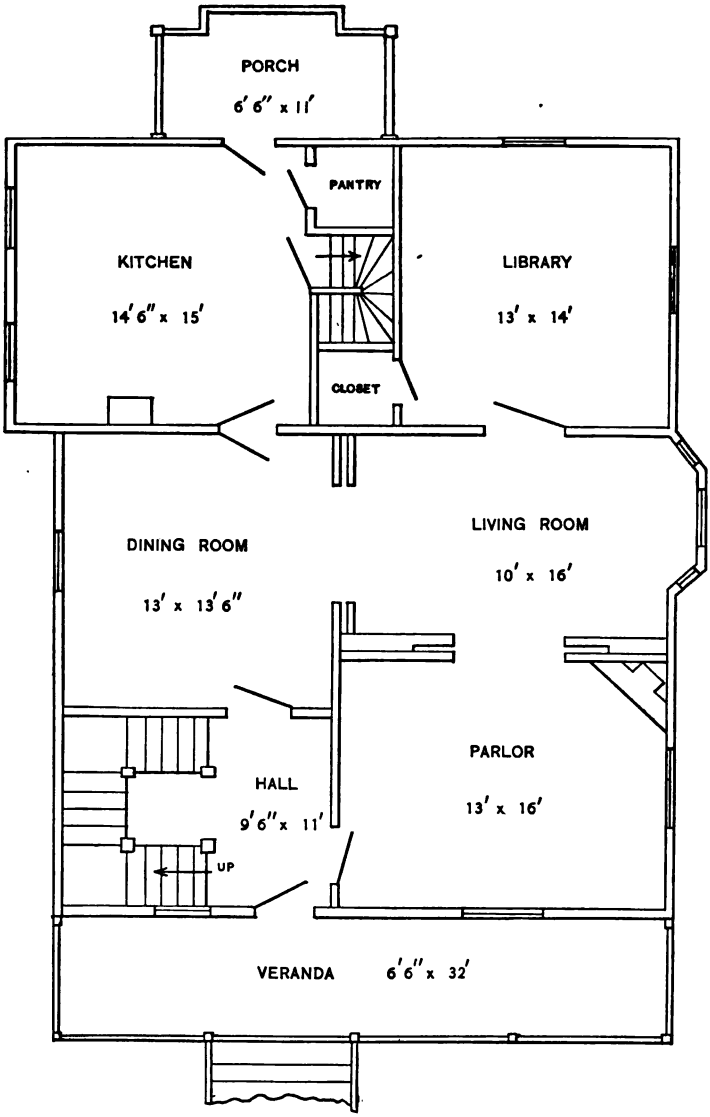
2. How many square yards in the walls and ceiling of a room 18 by 20 ft. and 9 ft. high? How many laths, and how much stucco will it take to cover it?

3. A hall is 5 × 15 ft. and 10 ft. high. What will it cost to lath and plaster it at 25¢ a square yard? How many laths will it take?

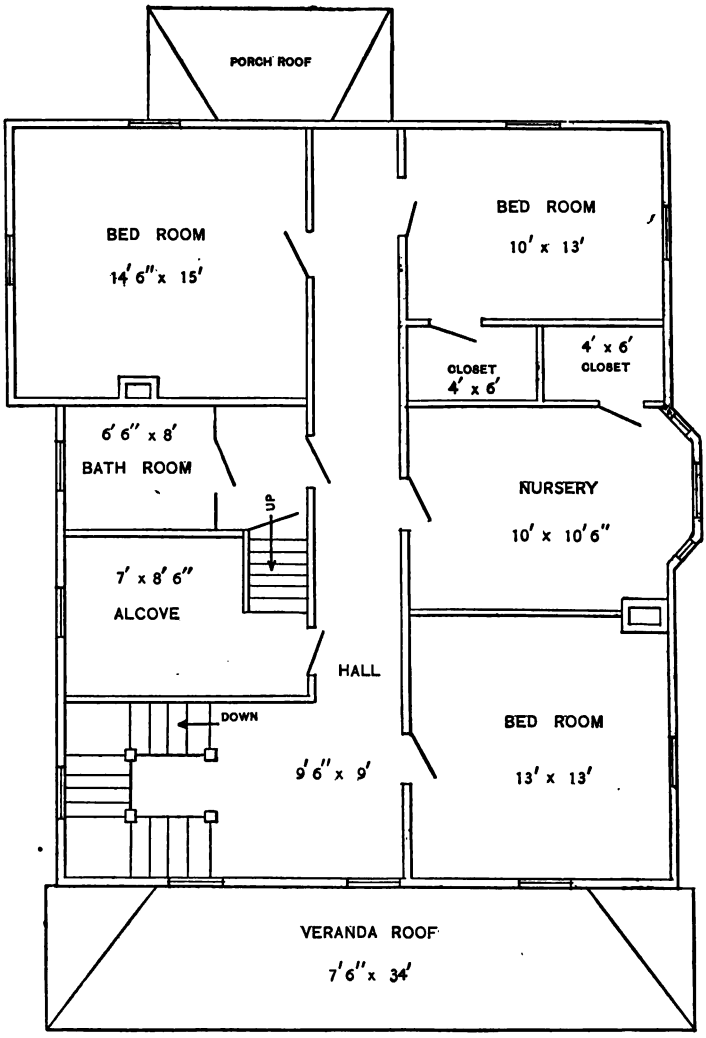
FLOOR PLANS OF HOUSE



BASEMENT


PORCH
6′ 6″ x 11′
PANTRY
KITCHEN
14′ 6″ x 15′
LIBRARY
13′ x 14′
CLOSET
DINING ROOM
13′ x 13′ 6″
LIVING ROOM
10′ x 16′
PARLOR
13′ x 16′
HALL
9′ 6″ x 11′
UP
VERANDA
6′ 6″ x 32′

First Floor



SECOND FLOOR

4. A dining room is 18 by 20 ft. and 11 ft. high. At 45¢ per square yard what will it cost to lath and plaster the walls and ceiling?

5. A bedroom is 12 ft. square and 10 ft. high. What will it cost to plaster it at 30¢ per square yard?

6. The rooms in the floor plan on pp. 90–91 are 11 ft. high. Making allowance for doors and windows as suggested in the discussion, what will it cost to plaster the house, if sand costs 75¢ per cubic yard, lime $1.25 per barrel, and hair 25¢ per bushel, and the cost of putting it on is 20¢ per square yard?

7. What would it cost to lath, and plaster the house with stucco (900 pounds to the 100 square yards), if laths are worth 50¢ a bundle, stucco $10.50 per ton, and the cost of putting on the laths and stucco is 25¢ per square yard?

65. Papering. Wall paper is put up in single rolls 8 yards long, and double rolls 16 yards long. English rolls are single rolls only, and are 12 yards long. The width of ordinary or figured paper is 18 inches, that of ingrain is 30 inches. Higher grades vary somewhat in width, from 18 to 22 inches, and from 30 to 36 inches. Border and friezes are sold by the linear yard. They vary in width.

In a roll of 18-inch paper, there would be $1\frac{1}{2}$ rows of 24 sq. ft., or 36 sq. ft. In 30-inch paper, there would be $2\frac{1}{2}$ rows of 24 sq. ft., or 60 sq. ft. In 22-inch paper, there would be $1\frac{5}{8}$ rows, or 44 sq. ft., etc.

The surface to be papered in a room, 18 × 22 ft., 12 ft. high, would be as follows: In the four walls there would be 12 rows of 80 (the perimeter of the room) sq. ft., or 960 sq. ft., and in the ceiling 18 rows of 22 sq. ft., or 396 sq. ft. In finding the number of rolls necessary to cover the surface, the waste due to matching must be considered, and allowance made for windows and doors. The latter is usually 20 sq. ft. for each opening, counting archways and double doors as double openings. As whole rolls must be bought, it often happens that there is a part of a roll remaining, sufficient, perhaps, to cover waste due to matching, after the exact

surface has been provided for. If a border were used, its width would be deducted from the height of the room.

If there were two doors and three windows in the above room, 100 sq. ft. would be deducted, leaving 860 sq. ft. in the walls. There being 36 sq. ft. in a roll, $23\frac{8}{9}$ rolls would cover the surface; 24 rolls, at least, would be required ($\frac{1}{9}$ roll only being allowed for matching). Whether this would exactly cover the surface would depend upon the pattern. For the ceiling, 11 rolls would exactly cover the surface, and 12 would probably be necessary to provide for waste in matching. The border would be 80 ft. or 27 yd. long.

PROBLEMS

1. A room is 20 by 24 ft. and 10 ft. high. What will it cost to paper it with 18-inch paper (always use single rolls unless otherwise mentioned) at 50¢ a roll, if there are two doors and three windows?

2. A dining room is 18 by 24 ft. and 11 ft. high. What will it cost to paper it with paper 22 in. wide, worth 90¢ a double roll, if there are two doors and four windows?

3. How many rolls of paper 30 in. wide will it take to paper the parlor in the plan on page 90, if a border 22 in. wide is used?

4. How many rolls, 36 in. wide, to cover the dining room (p. 90) if a border 18 in. wide is used? Cost at $1 a roll for body, 60¢ a yard for border, and 65¢ a roll for ceiling?

5. Find cost of papering the library (p. 90) with paper 22 in. wide, at 95¢ a roll, and a border 18 in. wide at 95¢. Deduct for a wainscoting 3 ft. high all around the room.

6. Find cost of papering the kitchen (p. 90) with 18-inch paper at 45¢ a roll.

7. Find cost of papering the halls of the house (pp. 90–91), deducting for the stairway and doors, with 22-inch paper costing $1.10 a double roll. Use a border of the same width costing 30¢ a yard, and 18-inch paper for ceiling at 40¢ a roll.

8. Find cost of papering the bedrooms (p. 91) with 18-inch paper at 35¢ a roll, using a border of the same width and costing 20¢ a yard.

LAND MEASURE AND SURVEY

66. The Township. For purposes of public record, so that land ownership may be secure and transfers of title conveniently and safely made, a system of land survey is necessary.

In the United States prior to the year 1785, each tract of land was surveyed and its boundary line described as starting from some natural object, thence a certain distance in some direction, thence in another direction to another object, etc. The results of such surveys were irregular shapes of land tracts and often endless litigation. This method is still in use in the Eastern states.

By an ordinance of the Continental Congress, a system of rectangular survey was adopted in 1785 for the Northwest Territory, and later this was applied to all the Central and Western states.

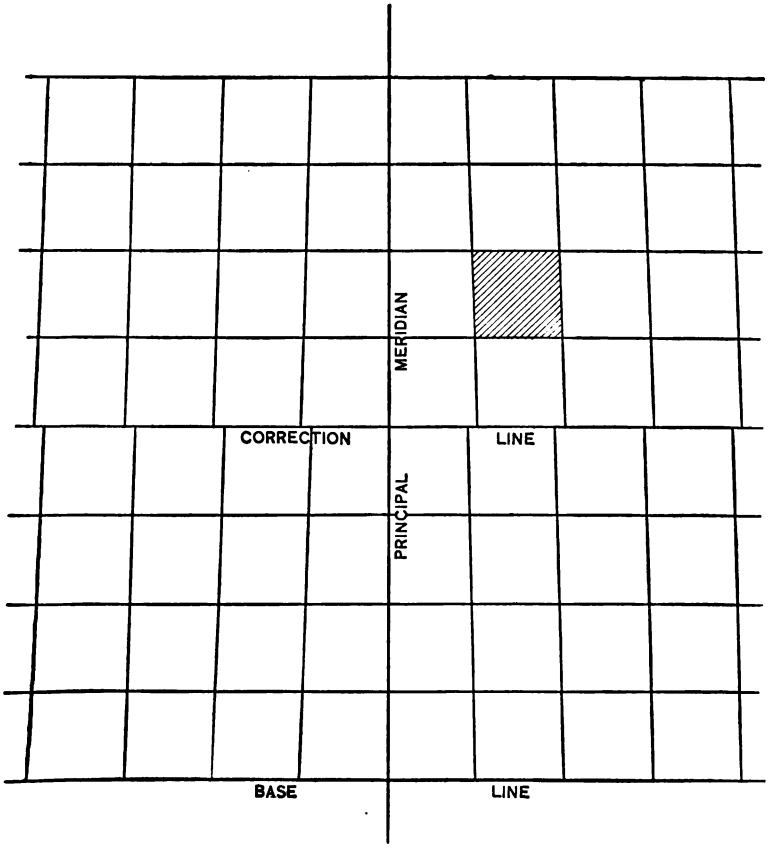
By this system, the land is surveyed into rectangular *townships*, six miles square, with boundary lines conforming to the points of the compass. To locate a particular township certain meridians are designated as *principal meridians* and certain parallels as *base lines* or *standard parallels*. Twenty-four principal meridians have been established, the first one being in Ohio and the last one in Oregon. Parallels most convenient in the survey of a state are selected for base lines.

The townships are numbered, in order, north and south of the base line, and the particular row of townships is shown by the number of rows or *ranges* east and west of the principal meridian. Thus, T 7 N, R 4 W, would mean the seventh township north of the given *base line*, in the fourth range of townships west of the given *principal meridian*.

Starting from the principal meridian, the surveyors mark off points every six miles on the base line. From these points, lines are run to the north by compass. Because the

meridians approach as they near the pole, it is clear that these north lines would soon be less than six miles apart as they extend northward. To preserve the townships as nearly uniform in size as possible, new parallels are run at frequent intervals as *correction lines*, upon which the north lines are erected again, starting six miles apart.

The shaded township in the accompanying illustration



GOVERNMENT SURVEY

would be described as Township 6 North, Range 2 East of the Principal Meridian.

67. Subdivisions of the Township. The surveying unit, the township, is six miles square, and contains 36 sq. mi.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

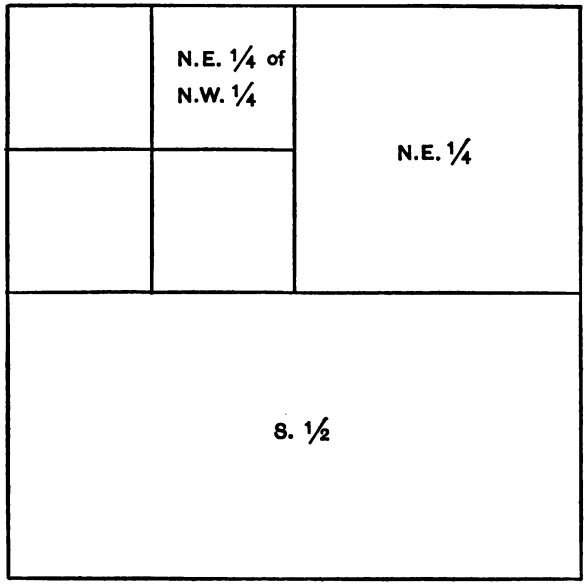
A Township

These square miles are called *sections*, and are numbered from 1 to 36, beginning at the northeast corner, going west on the northern row of sections, east on the second row, west on the third, etc.

Since (Sec. 53) there are 640 acres in a square mile or

section, in half a section there are 320 acres, and 160 acres in a quarter section. One half of the latter is called an "eighty." A tract of land within a section is located by the use of fractions with descriptive directions; thus, NW $\frac{1}{4}$ is the northwest quarter of the section, and the S $\frac{1}{2}$ of NW $\frac{1}{4}$ is the south half of the northwest quarter section.

LOCATION OF SECTIONS



A SECTION

A complete description of a tract of land might be: the NE $\frac{1}{4}$ of the NW $\frac{1}{4}$ sec. 24, T 12 N, R 9 E of the 5th Principal Meridian, which would be read as the northeast one fourth of the northwest quarter of section 24, township 12 North, range 9 East of the 5th Principal Meridian.

PROBLEMS

SUBDIVISION OF SECTIONS

1. Make a diagram of a township. Locate Sec. 20, 12, 9, and 13.

2. Make a diagram of a section. Locate the NW ¼, and S ½.

3. Mr. Brown owns 40 acres in the SE corner of section 13, township 5 N, range 3 E. Locate his land.

4. A man bought the E ½ of the NW ¼, the W ½ of the NE ¼, and the NE ¼ of the SE ¼. How many acres did he buy? Locate.

5. I sold the E ½ of a section of land for $24.50 per acre, the NW ¼ for $28, the E ½ of SW ¼ for $23.75, and the remainder of the section for $20. How much did I get for the entire section?

6. A man bought the following property: the SE ¼, the NE ¼, the N ½ of NW ¼, the NW ¼ of the SW ¼, and the E ½ of the SW ¼ of Sec. 16, T 13 N, R 3 E. How many acres did he buy?

MEASUREMENT OF NONRECTANGULAR SURFACES

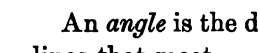
68. Surface Forms. A *surface* is the outside or face of a solid.

A *plane surface* is one that is flat. A line connecting any two points in such a surface will lie wholly within the surface. Being the outside or face merely, a surface is not considered as having thickness. It is said, therefore, to have but two dimensions, length and breadth.

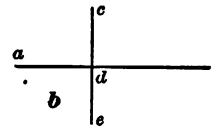
The edges of surfaces form *lines*. These lines may be *straight*, when their direction is never changed; or *curved*, when continually changing, so that no three adjacent points are in the same straight line.

An *angle* is the difference in direction between two straight lines that meet.

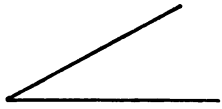
When two meeting lines go in exactly opposite directions, the angle formed is a *straight angle* (*a*). The angles formed on either side of (*a*), by lines *ab* and *ac*, leaving the point *a* in exactly opposite directions, are straight angles.

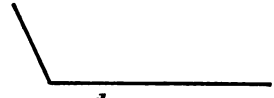


A *right angle* is one half a straight angle. When one line meets another so as to form two equal angles, as *ade* and *adc*, the angles formed are right angles (*b*).

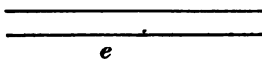


An *acute angle* is less than a right angle, and an *obtuse angle* is greater than a right angle (*c* and *d*).


c

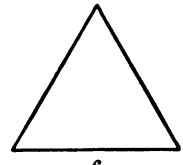

d

Lines are *parallel* when they extend in the same direction and are at all points equally distant from each other (*e*).

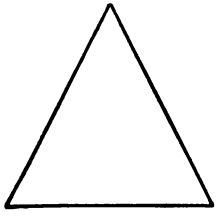

e

Lines are *perpendicular* when the angles formed by their meeting are right angles (*b*); *horizontal* when parallel to any line connecting two points on the horizon of vision (*e*); *vertical* when perpendicular to a horizontal plane (*b*).

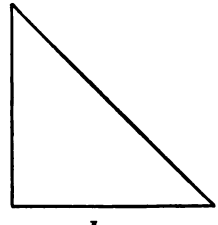
A *triangle* is a plane surface bounded by three straight lines.


f

A triangle is *equilateral* when its three sides are equal (*f*); *isosceles* when two of its sides are equal (*g*), and *scalene* when all three sides are unequal (*i*).

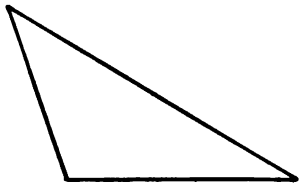

g

A *right triangle* has one right angle (*h*), an *obtuse triangle* has one obtuse angle (*i*), and an *acute triangle* has three acute angles (*f*).


h

A *polygon* is a plane surface of more than four sides and bounded by straight lines.

The aggregate length of all the lines bounding a polygon is called its *perimeter*.



i

A *quadrilateral* is a plane surface bounded by four straight lines.

A *parallelogram* is a quadrilateral the opposite sides of which are parallel (*j*).

A *trapezoid* is a quadrilateral two sides of which are parallel (*k*).

A *trapezium* is a quadrilateral with no two sides parallel (*l*).

j

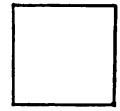
k

A *rectangle* is a parallelogram the angles of which are right angles (*m*).

A *square* is a rectangle, the sides of which are equal (*n*).

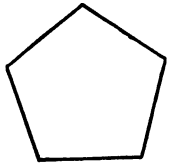
l

m

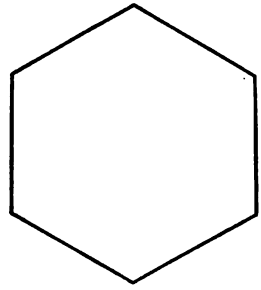


n

Other polygons are named likewise from the number of sides, *i.e.* pentagon (*o*), hexagon (*p*), octagon, decagon (five, six, eight, ten), etc.

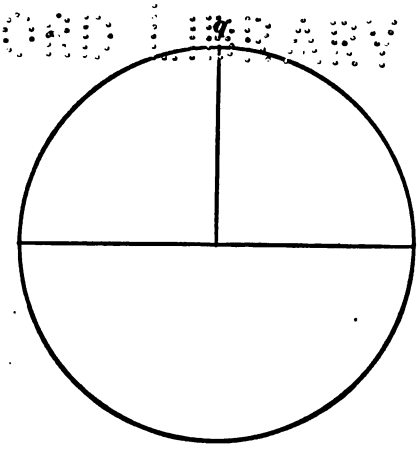


o



p

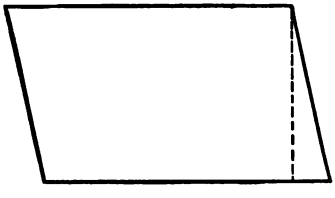
A *circle* is a plane surface, every point in the boundary of which is equidistant from a point within called the *center*. The curved line forming the boundary of a circle is called the *circumference*. The distance from a given point on the circumference through the center to the opposite point is the *diameter*. One half this diameter is the *radius* (q).



QUADRILATERALS

69. Oblique Parallelogram. Take a piece of paper and make a rectangle 3 by 5 in. Mark off one inch along the upper edge from the right upper corner. Draw a line from this mark to the right lower corner. Fold along this line, crease, and tear off.

Along the lower edge or base, mark off 1 in. from the left lower corner, and draw a line to the left upper corner, crease, and tear. The resultant figure is an oblique parallelogram. Why a parallelogram? Why oblique?

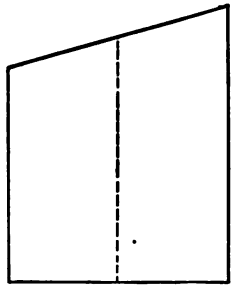


Again, on the lower base, mark off 1 in. from the right lower corner, and draw a line to the right upper corner. Crease and tear. Place the triangle to the left edge of the parallelogram so that the oblique edges meet. The oblique parallelogram has been changed into a rectangle. Has the base been changed?

The altitude? The perpendicular height of a figure is its *altitude*.

An oblique parallelogram is equivalent to a rectangle with the same base and altitude.

70. Trapezoid. Cut out a 5-inch paper square. Two inches to the left of its right upper corner draw a line to the right lower corner. Fold along this line, crease, and tear. The four-sided piece is a trapezoid. Why?



Fold the trapezoid so that the parallel sides will meet. Crease and tear. Turn the upper piece upside down and place it to the right of the lower one, so that the oblique edges will meet. What have you formed? Compare the base of the rectangle with the combined length of the two parallel sides (called bases) of the trapezoid. How does its altitude compare with that of the original trapezoid?

A trapezoid is equivalent to a rectangle with a base equal to the sum of the two bases of the trapezoid, and one half its altitude.

PROBLEMS

1. One side of a walk is 30 ft. long and the other 27½ ft. and the width 5 ft. How much will it cost to build it at 30¢ per square yard?

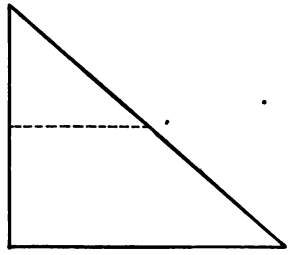
2. One side of a field in the shape of a trapezoid measures 180 yd., the other 200 yd. How many acres in the field if the perpendicular distance between the sides is 75 yd.?

3. What is the area of an oblique parallelogram, the perpendicular height being 24 ft. and the length 56 ft.?

TRIANGLES

71. Right Triangles. Take a piece of paper 4 inches square. Fold and cut along the diagonal. You now have two right

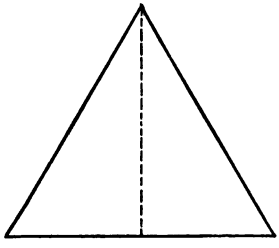
angles. Take one of them and fold so that the vertex of the upper angle will meet the vertex of the right angle. Crease and tear. Place the upper part upside down and with its oblique side next to the oblique side of the larger piece. What have you made?



How does the base of the rectangle compare in length with that of the triangle? The height of the rectangle with the altitude of the triangle?

A right triangle is equivalent to a rectangle on the same base with half the altitude.

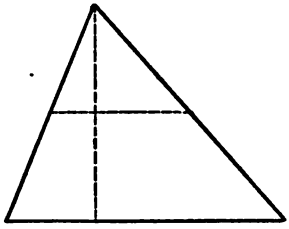
72. Isosceles Triangle. Take a 4-inch paper square. From the middle point of the upper edge draw lines to the lower corners. Fold along the lines, crease, and tear. The large triangle remaining is an *isosceles* triangle. Why?



Fold the upper vertex down so it will just touch the middle point of the base. Crease and tear. Connect the middle points of the two bases by a straight line. Fold along this line, crease, and tear. Turn the two pieces upside down and place one on each side of the triangular piece, with oblique edges together. What have you formed? Compare base of the triangle with that of the original triangle. Compare their altitudes.

An isosceles triangle is equivalent to a rectangle on the same base and with half the altitude. As an equilateral triangle is isosceles, the same would hold true.

73. Scalene Triangle. Cut a paper triangle, no two sides of which are equal, and not a right triangle. Fold over the vertex to meet the base, and so that the line of the fold will be parallel with the base. Crease and open. Fold again along a line drawn perpendicular from the vertex to the base. Crease and tear. Tear both new triangles at first fold. Place the oblique sides of the two parts of the new triangles together. Then place the two resultant rectangles side by side. Compare the entire base with that of the original triangle. Compare their altitudes. *A scalene triangle is equivalent to a rectangle on the same base and with one half the altitude.*



As the same truth is seen, in the above, to hold good for equilateral, isosceles, and scalene triangles, whether right, obtuse, or acute-angled, the general statement may be made that *any triangle is equivalent to a rectangle on the same base and with half the altitude.*

74. Other Polygons. All other polygons may be resolved into rectangles or triangles. Finding the area of the parts, the whole may be found. For example, among the more common geometrical figures, a trapezium may be resolved into two triangles; a hexagon and octagon may be resolved into six and eight triangles focusing at the center.

PROBLEMS

1. How many square inches in a right triangle, if the base is 18 in. and the altitude 24 in.?

2. The base of an isosceles triangle is 12 ft. and the altitude is 16 ft. What is the area?

3. How many square feet in a scalene triangle, with the base 15 ft. and altitude 20 ft.?

4. A triangular field is 40 rd. along its base and 26 rd. along the altitude. What is it worth at $72 per acre?

5. What is the area of a hexagon composed of equilateral triangles, each side of which is 8 in.? (Use the nearest inch as altitude.)

6. What is the cost of painting a church steeple, the base of which is an octagon 8 ft. on each side, and whose slant height (altitude) is 90 ft., at 25¢ per square yard?

7. I have a triangular lawn that I wish to sod. The base is 25 yd. and the altitude is 24 yd. What will it cost at $1.25 per square rod?

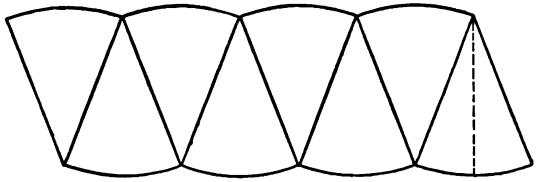
8. A hexagonal silo is 12 ft. on a side and the distance from the middle point of a side to the center is 10.4 ft. What is the area of the bottom?

9. I have a plot of ground in the form of a trapezium. One of the diagonals measures 42 yd., and the altitudes of the two triangles measure 20 yd. and 18 yd. respectively. What is its area?

75. Ratio of Circumference of Circle to its Diameter. Procure a well-cut circular disk, or a wheel. Measure carefully its diameter. Mark distinctly some point on the circumference. Starting with that point at the end of a line drawn on some flat surface, roll the disk or the wheel along that line until the marked point on the circumference again touches the line. Measure the length of the line thus marked off. This will be the length of the circumference. Divide this length by the length of the diameter. The quotient or ratio will be found to be nearly $3\frac{1}{7}$. In other words, *the circumference of a circle is* $3\frac{1}{7}$ *times its diameter.* For greater accuracy, the ratio of 3.1416 is used. In mathematics, its symbol is the Greek letter π (pronounced pi). For ordinary purposes $3\frac{1}{7}$ is sufficiently exact.

76. Area of a Circle. Fold a rectangular piece of paper, 5 by 6 inches, so that the long edges meet; crease but do not open. Fold so that the short edges meet; crease but do not open. Fold so that the folded edges will meet, and crease.

Mark off two inches from the point on both the edges, and midway between the edges. Connect the three points thus made by a uniformly curved line. Cut through the paper on this line. Open the paper, and a circle is shown. Cut through all the creased lines, forming eight equal *sectors*. Arrange seven of these sectors in a row, with alternate points and arcs along one side. Fold the oblique edges of the eighth sector together; crease and cut. Place a half on each end of the row. What have you approximately formed (see figure)? What part of the circumference forms the base? What is the altitude? If the sectors were sixteenths, the resultant would be more nearly a perfect rectangle; if thirty-seconds, still more nearly, etc. We may conclude, then, that:



THE SECTORS OF A CIRCLE

A circle is equivalent to a rectangle with one half the circumference as a base, and the radius as its altitude.

NOTE. — The geometrical formula for the area of a circle is πr^2, or the square of the radius multiplied by $3\frac{1}{7}$.

PROBLEMS

1. The area of a circle 14 in. in diameter with a circumference of 44 in. is equivalent to a rectangle with a base of —— inches ($\frac{1}{2}$ the circumference), and an altitude of —— inches (radius).

2. What are the dimensions of a rectangle that may be formed from a circle 7 in. in diameter and a circumference of 22 in.?

3. The diameter of a circle is 21 in., the circumference 66 in. Find the area.

4. Find the area of a circle with a diameter of 8 in.

OTHER SURFACES

If a paper 6 by 8 inches were rolled into a cylinder with the long edges meeting, what would be the circumference of the cylinder? What the length?

The surface of a cylinder is equivalent in area to that of a rectangle having the length of the cylinder as a base, and its circumference as the altitude.

The surface of a pyramid consists of the rectangular base and triangular sides. The triangles have a common altitude, or slant height, hence *the lateral area of a pyramid is equivalent to that of a rectangle having the perimeter of the pyramid as a base, and one half the slant height as the altitude.*

The lateral area, likewise, *of a cone is equivalent to that of a rectangle having the circumference of the cone as a base, and one half the slant height as the altitude.*

The surface of a sphere is equivalent to a rectangle having the circumference of the sphere for a base, and its diameter for the altitude.

GENERAL PROBLEMS

1. What is the area of the surface of a globe 6 in. in diameter?

2. What is the entire surface of a cylinder whose diameter is 6 in., height 2 ft.?

3. A cylindrical cistern is 10 ft. deep and 7 ft. in diameter. What will it cost to cement it, at 35¢ a square yard?

4. In a warehouse there are six hoppers of pyramidal shape, 6 ft. high and 8 ft. square. How many square feet on their surfaces?

5. What will it cost, at 24¢ per square yard, to paint a spire of conical shape, if the slant height is 75 ft. and the diameter of the base is 16 ft.?

6. A rectangular park is 80 by 160 ft. In the left- and right-hand corners at one end are flower beds which are 5 ft. wide, and extend 15 ft. both ways from the corners. In the left-hand corner at the other end is a triangular bed, the altitude of which is 12 ft. and the base 16 ft. In the right-hand corner is a bed in the form of a trapezoid 7 ft. broad, whose sides are respectively 24 and 18 ft. In the center is a pond 42 ft. in diameter, with a walk 3 ft. wide extending from one side of the park

to and around the pond. The remaining space is a lawn. How many square feet in the lawn? In each bed? In the pond? In the walk?

7. A street in a city is 100 ft. wide and 1 mi. long. What will it cost to have it paved with brick, 4 by 8 in., at $6.20 per M.?

8. A horseman has a field 20 rd. square in which he puts a circular track 50 ft. wide. What is the area of the track? Its length?

9. A man owned a lot 80 rd. square. A corporation bought the right of way for an electric line at $195 per acre. One fence inclosing the track was to commence at the northwest corner of the lot and run to a point 20 rd. east of the southwest corner; the second fence to commence at a point 15 rd. south of the northeast corner and run to a point 12 rd. east of the first. What did the right of way cost? How many posts, set 8 ft. apart, will it take for the fences? Find cost of the fencing at 65¢ a rod.

10. A room is 15 by 18 ft. How many yards of carpet, 27 in. wide, will it take to cover it, allowing 9 in. for matching?

11. A building is 40 by 80 ft. and 15 ft. high. The altitude of the gable ends is 14 ft., and the roof projects 2 ft. over each side and end. What will it cost to cover this roof with shingles, laid 4½ in. to the weather, at $4.50 per M? What will it cost to paint the roof and the body of the building, at 25¢ per square yard?

12. A buys a piece of land 20 rd. square, paying $250 an acre. He divides it into lots 46 by 100 ft., after deducting for a street 50 ft. wide around the piece and an alley 30 ft. wide through the center. The expense of setting out trees, grading, etc., was $754.75. If he sold the corner lots for $250 apiece, and the others for $200 apiece, what did he gain by the transaction?

13. A house has 8 rooms, with dimensions as follows: 3 rooms 15 by 18 ft., 2 rooms 10 by 14 ft., and 3 rooms 12 by 16 ft., and 8, 9, and 10 ft. high respectively. There are 24 windows and 8 doors. Find the cost of lath to cover them at 30¢ a bundle. How much lime, sand, and hair will it take to plaster the house? How much stucco would it take, counting 900 lb. to 100 square yards?

14. The first three rooms in Problem 13 are to be covered with 18-inch paper, the next three with 22-inch paper, and the last two with 30-inch paper. Allowing 20 sq. ft. for openings, how many rolls of each kind of paper will it take to cover the rooms, if the doors and windows are divided equally among the rooms?

VIII

MEASURES OF VOLUME

77. Units of Volume. *Volume* is the quantity of space occupied. Any given space has length, breadth, and thickness. Therefore volume has to do with these three dimensions. Volume is measured by *cubic units*, *i.e.* units whose length, breadth, and thickness are the same.

A rectangular body whose dimensions are equal is called a *cube*. These cubic units correspond to units of length, viz. a cubic inch, a cubic foot, and a cubic yard.

The standard unit is the *cubic yard*. From the fact that volume is measured by cubic units, measures of volume are called *cubic measure*.

Table of Cubic Units

There are:

1728 cubic inches (cu. in.) in 1 cubic foot (cu. ft.).
27 cubic feet in 1 cubic yard (cu. yd.).

EXERCISES

1. Procure or make a number of inch cubes. Through touch and sight, seek to develop an accurate concept of a cubic inch. Cut, without measurement, an inch cube from any available material. Test its accuracy. Practice.

2. Practice estimating the volume in cubic inches of a box, a book, the desk top, a piece of wood, etc. Use a ruler for testing the accuracy of your judgment.

3. By pasting the edges of six one-foot squares of pasteboard together, a cubic foot may be made.

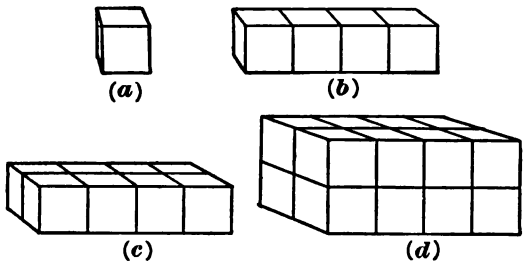
4. Develop a concept of the one-foot cube in the same manner as above.

5. Estimate the volume in cubic feet of large boxes, rooms, halls, foundations of buildings, etc. Test correctness with a ruler. Practice for accuracy in judgment of the number of cubic feet in different volumes.

6. Measure off a cubic yard from the corner of a room. Fix as accurate a concept of it as possible.

7. Estimate volume of various solids in cubic yards. Test in practice.

78. Finding Volume. Finding the volume of a solid consists in finding the cubic units of space occupied by it. In finding the volume of rectangular solids, we first think or image the cubic unit, then the number of these units along one edge or in one row, then the number of such rows forming a layer, and finally the number of layers of cubic units in the solid. The number of cubic units in one row is the same as the number of linear units along an edge or one dimension; the number of such rows is the same as the number of linear units along an edge forming another dimension; and the number of layers is equal to the linear units along the edge forming the third dimension. We say, then, that the length indicates the number of units in one row, the breadth the number of rows, and the thickness the number of layers.



(*a*) A cubic unit. (*b*) A row of four cubic units. (*c*) A layer of 8 cubic units. (*d*) A solid of 16 cubic units.

To find the volume of any rectangular solid, *multiply the number of cubic units in one row by the number of rows forming one layer, and that product by the number of such layers.*

Thus, a solid 6 in. long, 5 in. wide, and 4 in. high has 6 cu. in. in each row, 30 cu. in. in five rows, or one layer, and 120 cu. in. in the four layers of the solid. This may be expressed as follows:

$$6 \text{ cu. in.} \times 5 \times 4 = 120 \text{ cu. in.}$$

PROBLEMS

1. A box is 10 by 20 inches and 4 inches deep. How many cubic inches in each layer? How many in the box?

2. A box is 6 feet long, 4 feet wide, and 4 feet deep. How many layers of cubic feet? How many cubic feet in the box?

3. How many cubic feet of water will a tank hold if it is 12 by 6 feet and 3 feet deep?

4. What will it cost to dig a cellar 22 by 16 feet and 10 feet deep, at $1.35 a load (cubic yard)?

5. How many cubic feet in a piece of timber 6″ × 8″ × 40′?

6. How many cubic feet of wheat in a bin 6 by 12 by 8 feet high?

7. A schoolroom is 60 feet long, 40 feet wide, and 12 feet high. If there are 120 pupils, how many cubic feet of air to each pupil? How often must the air be changed to give fresh air, if a pupil requires 2400 cubic feet of air per hour?

8. How many cubic yards of earth must be removed in digging a trench for the walls of a building, if the building is 40 by 100 feet, the walls 2 feet wide, and the trench 3 feet deep?

9. I wish to build a road. The grade would cost $80 a mile. The gravel is to be spread 9 inches deep and 16 feet wide. What will the road cost per mile, if the gravel is worth $2 per cubic yard?

10. My lawn is 70 by 120 feet. What will be the cost for dirt to raise it 18 inches at 50¢ a load?

11. The abutments of a bridge are 30 feet long and 6 feet wide and 15 feet high. Find the pressure exerted, counting 160 pounds to the cubic foot.

12. A man has a pond 5 by 20 rods. The ice is 9 inches deep. Counting $\frac{1}{4}$ of the space for sawdust, and $\frac{1}{10}$ loss of ice in handling, what will be the height of a building 80 feet long and 40 feet wide that will hold it?

79. Cubes and Roots. A cube as defined in Sec. 77 is a rectangular body whose three dimensions are equal. That is to say, the number of cubic units along one edge is the same as the number of rows and the number of layers of such units. In finding the volume of such a body, the same number will, therefore, be used as a factor three times, and the resultant volume is said to be expressed by the cube of the number, or the number is said to be raised to the third power. Thus, the cube of three may be expressed: 3 cubic units $\times 3 \times 3 = 27$ cubic units, or, without reference to definite units; as, $3 \times 3 \times 3 = 27$. This may also be expressed as $3^3 = 27$.

Finding the volume of a cube when the number of cubic units along one edge only is given, is called *cubing* a number. When the volume is known, and the problem is to find the number of cubic units along one edge, the process is known as finding the *cube root*. Thus, the cube of 2 is 8, or of 2 cu. ft. is 8 cu. ft. The cube root of 8 is 2, or the cube root of 8 cu. ft. is 2 cu. ft.

EXERCISES

1. The cube of three feet has how many cubes in each row? How many rows? How many layers? How many cubic feet in all?

2. The cube of 5 feet contains how many rows? How many cubic feet?

3. A cube of 9 inches has how many cubic inches?

4. A cube containing 125 cu. ft. has how many cubic feet along one edge?

5. A cube containing 27 cubic inches has how many cubic inches in each row? The cube root of 27 is?

6. The cube of 4 is? Of 6 is? Of 7 is? Of 8 is?

7. The cube of 10 is? Of 11 is? Of 12 is? Of 15 is?

8. The cube of 20 is? Of 25 is? Of 30 is? Of 50 is? Of 40 is? Of 75 is?

9. The cube root of 64 is? Of 27 is? Of 125 is?

10. The cube root of 1000? Of 1728? Of 1331? Of 3375?

11. The cube root of 8000? Of 15,625? Of 27,000? Of 1,000,000?

Memorize the following table:

$1^3 = 1$	$7^3 = 343$	$15^3 = 3375$
$2^3 = 8$	$8^3 = 512$	$20^3 = 8000$
$3^3 = 27$	$9^3 = 729$	$25^3 = 15{,}625$
$4^3 = 64$	$10^3 = 1000$	$50^3 = 125{,}000$
$5^3 = 125$	$11^3 = 1331$	$100^3 = 1{,}000{,}000$
$6^3 = 216$	$12^3 = 1728$	

APPLICATIONS OF VOLUME

80. Wood Measure. Volume measurement forms a convenient way of finding the amount of many things bought and sold in bulk. Among the simplest of its applications is wood measure. For this purpose a special unit is used. Wood used for fuel, when first prepared for sale, is usually cut 4 feet long. A pile of wood, 4 feet high and 8 feet long, is called a *cord*. There would be, then, 4 layers of 32 cubic feet, or 128 cubic feet in one cord. Rough timber cut in 4-foot lengths is called cordwood. A *cord foot* is 4 feet wide, 4 feet high, and 1 foot long, and contains 16 cubic feet. There are 8 cord feet in a cord.

81. Stone Measure. Stone is divided into two classes at the quarry, viz. *dimension stone* and *rubble*. The first consists of pieces which are quarried in regular shapes and to a fixed size. This class of stone is generally sold by the cubic foot.

Rubble is the waste from quarrying the larger stones, and includes pieces of various sizes and shapes. It is usually sold by the carload, or in small quantities by the perch or cord, and in some places by the ton.

In the wall, dimension stone is generally measured by the square foot. Rubble is measured by the perch, which varies

from 16 to 25 cu. ft., usually $24\frac{3}{4}$ cu. ft.; or the cord, of 128 cu. ft., or the cubic yard. About $\frac{1}{5}$ of the volume is usually allowed for mortar, which is commonly mixed in the proportion of 3 parts of sand to 1 of lime.

Walls of the same thickness and height are thought of as one long wall, using the outside measurements of the building to determine its length. No allowance is made for openings in estimating cost of labor, unless openings are many or very large, when $\frac{1}{2}$ the volume is sometimes deducted.

82. Bricklaying. In estimating the number of bricks for a given foundation or building, the walls are considered as one wall, with the outside perimeter of the four walls as the length. The height being known, the area of the side of such a wall is then found. If the wall is to be a single brick in thickness, $7\frac{1}{2}$ bricks will be required for each square foot; if 2 bricks thick, 15; and if 3 bricks thick, $22\frac{1}{2}$ bricks. In estimating the cost of labor, allowance for openings is not usually made unless these are many and large. When deduction is made for openings, it is for one half their volume.

The above estimates are based on the size of the common brick, which is about $2 \times 4 \times 8$ inches. As the bricks are laid flat, the exposed side of each brick would be 2×8 inches, or 16 square inches. Nine bricks, if laid without mortar, would be required for one square foot (16 sq. in. $\times$ 9 = 144 sq. in.). Allowing about $\frac{1}{5}$ for mortar, $7\frac{1}{2}$ bricks would be required.

Bricks are now made in such a variety of sizes that it is often necessary to figure the number to a square foot, from the known size of the brick to be used in the particular building.

From 1 to $1\frac{1}{2}$ barrels of lime, according to quality, and $\frac{5}{8}$

of a cubic yard of sand are required for each 1000 bricks. No allowance, as a rule, is made for openings in estimating labor. Four bricklayers and one tender will lay from 7200 to 8000 bricks per day.

PROBLEMS

1. How many cords of wood in a pile 24 ft. long, 8 ft. high, and 4 ft. wide?

2. How many cords of stone in a wall 8 by 60 ft. and 2 ft. thick?

3. What will a pile of wood 4 by 6 by 48 ft. cost, at $4 per cord?

4. What will it cost to build the stone foundations for a barn, 20 by 40 ft., if the wall is 6 ft. high and 2 ft. thick, at $15 per cord?

5. If a cellar is 18 by 36 ft. and the walls 10 ft. high, how many bricks will it take to build it two bricks thick?

6. How many bricks would be necessary for the foundations of a building 25 by 30 ft., if it is 4 ft. high and 3 bricks thick?

7. The foundation of a house is 3 bricks thick and 3 ft. high. The total length, if extended, is 175 ft. How many bricks will it take? How much lime and sand? What will be the cost of the bricks at $9.50 per M? How long will it take four bricklayers and one tender to lay the foundation?

8. Commencing at the northwest corner, a wall extends east 30 ft., then south 10 ft., then west 10 ft., then south 10 ft., then east 10 ft., then south 10 ft., then west 15 ft., then north 5 ft., then west 15 ft., then north to the point of starting. The wall is 3 bricks thick and 4 ft. high. How many bricks will it take to lay it? How much sand and lime?

9. In the floor plan on p. 88, how many cords of stone would it take to lay the foundation 18 in. thick and 7 ft. high? How many cubic yards? How many perch of 24¾ cu. ft.? How much lime and sand would it take? How many bricks would it take for a wall 3 bricks thick? What will be the cost of brick at $8.75 per M? What will be the cost of lime and sand for either brick or stone wall, the lime at $1.00 per barrel; sand at 75¢ a load of one cubic yard?

83. Board Measure. Lumber is measured by the *board foot.* One square foot of surface on a board one inch thick

or less is called a board foot. A board foot is added for each added inch in thickness.

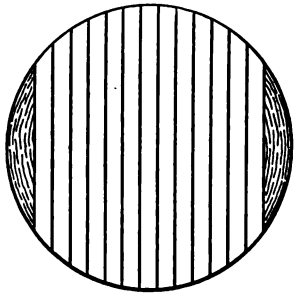
According to a method used by large dealers, the board 12′ long and 12″ wide is used as a basis for computation. The total number of board feet in the given number of boards is increased or decreased by such a fraction of the result as the length is of 12′ and the width is of 12″. A board 12′ long and 12″ wide would contain, manifestly, 12 board feet. A board 6″ wide would contain $\frac{1}{2}$ as much, or 6 board feet; 8″ wide, $\frac{2}{3}$ of 12 board feet, or 8 board feet, etc.

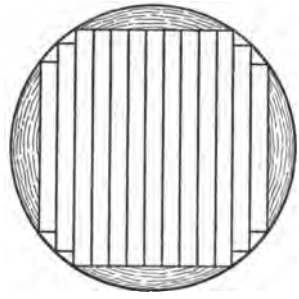
Twelve-foot boards, then, contain as many board feet as the board is inches wide. Thus, 24 such boards 8″ wide would contain 24 times 8 board feet, or 1092 board feet; if 14-foot boards, they would contain $\frac{1}{6}$ more, or 1274 board feet; if 16′ long, $\frac{1}{3}$ more, or 1456 board feet. The width of boards is counted as inches and half inches. If a board is not an even size, it is considered as the next smaller half inch.

84. Log Measure. For estimating the amount of lumber that may be cut from a given log, or for estimating the value of logs to be sold, it is necessary first to find a mean diameter. This is found by taking $\frac{1}{2}$ of the sum of the diameters of the two ends of the log, inside the bark.

Slabs are taken off to get the log into rectangular form and wide enough to be of use. 4″ of the diameter (2″ on each of the four sides) is deducted for slabs, and $\frac{1}{4}$ of the solid contents for waste in sawing (kerf) and in edging. Thus, a log with an average diameter of 14″, 12′ long, would form a rectangular log 10″ by 10″, 12′ long, and would contain 100 board feet. Allowing $\frac{1}{4}$ for kerf and other waste, the board product would be 75 board feet. If the length of the log was 16′, there would be $\frac{1}{3}$ more, or 100 board feet. Computations for other lengths are made in the same way. (Sec. 83.)

The accompanying diagrams show two of the many ways of sawing logs.





85. Lumberman's Reference Table. The following table, similar in form to the Lumberman's Reference Table, may be used for drill in finding mentally the number of board feet in boards, joists, etc., of various dimensions, using the table for verifying results. Notice that if the answer contains a fraction less than $\frac{1}{2}$, it is dropped, while if it is more than $\frac{1}{2}$, another foot is added.

PROBLEMS

1. How many board feet in 50 boards 8 inches wide and 12 feet long?
2. How many board feet in 60 boards 10 inches wide and 12 feet long?
3. How many board feet in 10 boards 2×4–12?
4. How many board feet in 20 boards 3×6–16?
5. How many board feet in 60 boards 1×10–20?
6. How many board feet in 10 boards 6×6–24?
7. The Koontz Lumber Company sold the following bill of lumber: 20 pc. $2 \times 6 - 10$, 15 pc. $4 \times 4 - 14$, 25 pc. $4 \times 4 - 16$, 40 pc. $2 \times 6 - 14$, 75 pc. $2 \times 10 - 8$ at \$20 per M; 100 pc. $2 \times 10 - 20$, 50 pc. $2 \times 6 - 22$, 45 pc. $2 \times 8 - 18$, 10 pc. $2 \times 12 - 16$, 75 pc. $2 \times 8 - 18$, at \$22 per M; 2050 posts at \$24 per C; 25 pc. $6 \times 8 - 24$ at \$20 per M; 125 bunches shingles at \$4.50 per M. Find the amount of the bill.

Reference Table

BOARD MEASURE, JOISTS, SCANTLING, AND TIMBER										
Size	12	14	16	18	20	22	24	26	28	30
2×4	8	9	11	12	13	15	16	17	19	20
2×6	12	14	16	18	20	22	24	26	28	30
2×8	16	19	21	24	27	29	32	35	37	40
2×10	20	23	27	30	33	37	40	43	47	50
2×12	24	28	32	36	40	44	48	52	56	60
3×6	18	21	24	27	30	33	36	39	42	45
3×8	24	28	32	36	40	44	48	52	56	60
3×10	30	35	40	45	50	55	60	65	70	75
3×12	36	42	48	54	60	66	72	78	84	90
3×14	42	49	56	63	70	77	84	91	98	105
4×4	16	19	21	24	27	29	32	35	37	40
4×6	24	28	32	36	40	44	48	52	56	60
4×8	32	37	43	48	53	59	64	69	75	80
6×6	36	42	48	54	60	66	72	78	84	90
6×8	48	56	64	72	80	88	96	104	112	120
6×10	60	70	80	90	100	110	120	130	140	150
8×8	64	75	85	96	107	117	128	139	149	160
8×10	80	93	107	120	133	147	160	173	187	200
8×12	96	112	128	144	160	176	192	208	224	240
10×10	100	117	133	150	167	183	200	217	233	250
10×12	120	140	160	180	200	220	240	260	280	300
12×12	144	168	192	216	240	264	288	312	336	360
12×14	168	196	224	252	280	308	336	364	392	420
14×14	196	229	261	294	327	359	392	425	457	490

8. How many board feet in a log 2 ft. in diameter at one end and 2′ 6″ at the other, and 12 ft. long?

9. A tree is 60 in. in circumference at the base and 48 in. at the top, and 42 ft. high. How many feet of lumber may be sawed from it?

10. A man built a close board fence 4 ft. high around a lot 120 yd. square. The boards were nailed in upright position to two railings 2 by

4 in., and the posts were set 8 ft. apart. He paid $20 per M for the railings, $30 per C for the posts, and $25 per M for the boards. What did the material cost?

11. How many feet of lumber will it take to board a barn 40 by 80 ft. and 25 ft. high, if the distance from the girders to the ridge of the roof is 18 ft.? Cost at $32 per M?

12. A lot 50 by 160 ft. is inclosed by a picket fence. The pickets are 4 ft. long, 3 in. wide, and 1 in. thick, and are placed 3 in. apart. The posts are placed 8 ft. apart, 2 × 4's being used at top and bottom for railing, and a baseboard 10 in. wide. What will the fence cost, if the posts are $20 per C, the lumber $22 per M, and pickets $4.25 per C?

13. Mr. A. H. Ward buys the following bill for a small house: 1 pc. 6 × 8 – 20, 2 pc. 2 × 8 – 26, 1 pc. 2 × 8 – 20, 4 pc. 2 × 8 – 18 @ $26; 45 pc. 2 × 8 – 14, 34 pc. 2 × 8 – 12, 44 pc. 2 × 6 – 12, 38 pc. 2 × 6 – 12, 3 pc. 2 × 6 – 18 @ $23; 130 pc. 2 × 4 – 18, 30 pc. 2 × 4 – 16, 30 pc. 2 × 4 – 14, 50 pc. 2 × 4 – 12 @ $28; 1700 ft. # 1 sheathing, 10,000 ft. # 1 fence flooring, 2000 ft. # shiplap, 2000 ft. # 2 fence flooring, 700 ft. straight grain flooring, 1360 ft. cedar siding @ $24; 320 ft. polished brass ceiling, 20 pc. 1 × 6 – 15, G. P. finishing @ $28; 5 rolls best paper, 2500 ft. @ $1.50 per C; 7 windows, 10 × 16 @ $1.80; 3 doors, 2 ft., 8 in. by 6 ft. 8 in. @ $5; 10 bbl. lime @ $1.15; 5000 hard brick @ $9.25 per M; 200 ft. 3½ in. crown molding @ $2.50 per hundred linear feet; 200 ft. 2-in. crown molding @ $2.50 per hundred linear feet. What is the amount of the bill?

MEASURES OF BULK

86. Dry Measure. For measuring small solids, such as grains, fruits, vegetables, etc., vessels are used whose volume or capacity is known. The volume of liquids must be obtained in the same way. Inasmuch as the solid form and the varying shape of grains, fruits, etc., leave unoccupied spaces between the solids, while liquids are compact, different unit measures have been established. The units for measuring solids form what is known as dry measure.

The established unit for dry measure is the *bushel*. It is usually cylindrical in form, 18½ inches in diameter, and 8 inches deep, containing 2150.42 cubic inches. For grain, shelled corn, etc., these measures are filled to the level, called

"stricken measure." For apples, potatoes, ear corn, etc., the measure is "heaped," and the bushel is supposed to contain 2747.71 cubic inches.

For finding the capacity, in bushels, of a grain bin, granary, or elevator, the bushel expressed in cubic feet is more convenient. A cubic foot (1728 cu. in.) is slightly more than .8 of a bushel (2150.42 cu. in.). To find the capacity of a bin, then, multiply .8 bushel by the number of cubic feet in the bin. For greater accuracy add $\frac{1}{5}$ of a bushel for each 100 cubic feet. If capacity in heaped bushels is desired, multiply .63 bushels by the cubic feet and correct as above for greater accuracy.

Thus, a bin 8 ft. wide, 10 ft. high, and 20 ft. long would contain 1600 cu. ft., and .8 bu. $\times$ 1600 = 1280 bu. Adding $\frac{16}{5}$ of a bushel, the contents would be 1285 bu.

To estimate the number of bushels of shelled corn in a crib of ear corn, if high grade and dry, take $\frac{1}{2}$ as many bushels as there are cubic feet.

TABLE

There are:	2 pints (pt.)	in 1 quart (qt.)
	8 quarts	in 1 peck (pk.)
	4 pecks	in 1 bushel (bu.)

NOTE. — The practical worth of a knowledge of dry measure consists not alone in knowledge of its units, but in power to estimate bulk in terms of units. Actual measures should be made in the classroom, and practice given in estimating bulk, verifying estimates.

For determining the number of bushels of different grains by weights see p. 139.

87. Liquid Measure. For measuring liquids, the standard unit is the *gallon*. It contains 231 cu. in. A cylindrical vessel 7 in. in diameter and 6 in. high contains a standard gallon.

For finding the capacity of cisterns, tanks, etc., the number of cubic inches may be divided by 231 cu. in. Since there are 1728 cu. in. in a cubic foot, there would be 7.4805+ gallons in a cubic foot. A shorter method for finding the number of gallons in a tank or cistern would be, then, to multiply $7\frac{1}{2}$ gallons by the number of cubic feet, and subtract $\frac{1}{400}$ ($\frac{1}{4}$ of $\frac{1}{100}$) of itself. This will give the correct result to a fraction of a gallon. Thus, a cistern $6 \times 8 \times 15$ ft. would contain 720 cu. ft. $7\frac{1}{2}$ gal. $\times$ 720 = 5400 gal. $\frac{1}{4}$ of $\frac{1}{100}$ of 5400 gal. = $\frac{1}{4}$ of 54 gal., or $13\frac{1}{2}$ gal. 5400 gal. $- 13\frac{1}{2}$ gal. = $5386\frac{1}{2}$ gal.

Casks, hogsheads, pipes, tuns, butts, tierces, carboys, etc., are indefinite standards, and their capacity is determined by measurement, and it is usually stamped on them. Barrels of oil and liquors are also indefinite. The barrel of $31\frac{1}{2}$ gal. was formerly a standard, and is still so used for some purposes.

TABLE

There are:	4 gills (gi)	in 1 pint (pt.)
	2 pints	in 1 quart (qt.)
	4 quarts	in 1 gallon (gal.)

88. Apothecaries' Liquid Measure. For measuring drugs, still smaller units are used. The gill is divided into fourths, known as fluid ounces. The size of small bottles is usually designated by the number of fluid ounces they contain. The further subdivisions are shown in the following table.

TABLE

There are:	60 minims (M)	in 1 fluid dram
	8 fluid drams	in 1 fluid ounce
	16 fluid ounces	in 1 pint
	8 pints	in 1 gallon

PROBLEMS

1. How many pints in 6 quarts? In 3 pecks? In 4 bushels?

2. How many pints in 8 quarts? In 5 gallons? In 6 gallons and 3 quarts?

3. How many quarts in 5 pecks? In 4 bushels? In 8 bushels 3 pecks? In 14 pecks?

4. What will 8 bu. 10 qt. cranberries cost at 12¢ a quart?

5. What will 3 pk. 5 qt. beans costs at 5¢ a quart?

6. Reduce 25 gal. 3 qt. 1 pt. to pints.

7. Change 196 pints to gallons.

8. Change 680 quarts to bushels.

9. Reduce $\frac{3}{4}$ bu. to pints.

10. How many fluid drams in 3 fluid ounces? How many in 3 pints? How many in 1 gallon?

11. What is the difference in the size in cubic inches of a quart dry measure and a quart liquid measure?

12. A bin for corn is $8 \times 9 \times 24$ ft. How many bushels of shelled corn can be obtained from the ear corn it would hold?

13. A swimming pool in a gymnasium is $15 \times 10 \times 40$ ft. How many gallons of water will it hold?

89. Forms of Solids. A *solid* is a body of matter or a defined portion of space.

A *solid* whose surfaces are planes is called a *polyhedron.* Polyhedrons are named tetra, hexa, octa, dodeca, or icosahedrons, as they have four, six, eight, twelve, or twenty faces.

A *prism* is a solid whose bases are equal, similar, and parallel, and whose sides are parallelograms. Prisms are described as triangular, square, oblong, etc., according to the form of their bases. They are right or oblique, as their sides are perpendicular or oblique to their bases.

A solid with two similar, parallel, and plane surfaces as bases, and curved surfaces as sides, is called a *cylinder.* If the bases are circles, it is said to be a circular cylinder. In speaking of a cylinder, a circular cylinder is usually meant.

A *cone* is a solid having a circle for a base, and tapering to a point or vertex.

A *sphere* is a solid or space contained within a given surface, every point of which is equidistant from a central point within.

MEASUREMENT OF NON-RECTANGULAR SOLIDS

90. Non-rectangular Prisms. In studying the measurements of area, all non-rectangular forms were reduced to equivalent rectangles. So in volume all solids not rectangular prisms should be considered as equivalent rectangular prisms, and then the volume computed, as in Sec. 78.

In all non-rectangular prisms, first determine the number of cubic units in one layer covering the base. If this base were rectangular, there could be placed upon it, in one layer, as many cubic units as there are square units in the area. By finding the area, then, of a rectangle equivalent to the base, we have the volume of one layer. The altitude or height of the prism is the number of layers.

The volume of one layer of cubic units on the base of a triangular, trapezoidal, or any other non-rectangular prism, is equal to the volume of one layer on a rectangle equivalent to such base, and there are as many such layers as the prism is corresponding units high.

91. Volume of Pyramids. Cut from cardboard four triangles with a 3-inch base and $5\frac{1}{4}$-inch altitude. Sew the edges together to form a pyramid, having an altitude of 5 inches.

Also cut four rectangles 3 by 5 in., and one 3 in. square; sew the edges of the rectangles together and to the edges of the square, to form a rectangular prism 5 in. long.

The prism and the pyramid thus formed have the same base, 3 in. square, and their common altitude is 5 in.

Fill the pyramid with dry sand, and empty it into the prism. It will require three times the contents of the pyramid to fill the prism. Its volume is, then, one third that of the prism.

The volume of a pyramid is equivalent to that of a rectangular prism with the same base and one third the altitude.

92. Cylinders. As in the prisms (Sec. 90), the number of cubic units on the base of a cylinder would be the same as on a rectangle equivalent to the area of the base. *The volume or capacity of a cylinder is equivalent to the volume of a rectangular prism whose base is one half the circumference in length and the radius in width, and whose height is the same as the length of the cylinder.*

93. Cones. Make from pasteboard, or procure a hollow cone, and a cylinder having the same base and altitude. Fill the cone with dry sand and empty it into the cylinder. Three times the contents of the cone will be found necessary to fill the cylinder.

The volume of a cone is equivalent to one third that of a cylinder with the same base and altitude.

94. The Sphere. A sphere may be considered as made up of many pyramids having their bases on the surface and their apexes at the center. Applying the principle of the pyramid (Sec. 91), the following statement is derived:

The volume of a sphere is equivalent to that of a rectangular prism with a base equal to the surface area of the sphere (Sec. 76) *and height equal to one third its radius.*

PROBLEMS

1. How many cubic inches in a prism 2 ft. high, the sides of the rectangular base being 10 and 16 in. respectively?

2. An ash hopper is in the form of a pyramid. If the base is 6 ft. square and the height 5 ft., how many bushels of ashes will it hold?

3. A cylindrical cistern is 12 ft. in diameter and 15 ft. deep. How many gallons of water will it hold?

4. A water filter in the form of a cone has a depth of 10 ft. The diameter of the base is 4 ft. How many bushels of charcoal will it hold?

5. The diameter of a cylindrical tank is 14 ft., the length is 18 ft. How many gallons of oil will it hold?

6. A man dug a well 4 ft. in diameter and 30 ft. deep. He got 4¢ a cubic foot and the dirt for digging. He sold the dirt for 50¢ a cubic yard. What did he get for his work?

7. A man has a bin 20 ft. long and 10 ft. high. If it is 4 ft. wide at the bottom and 6 ft. at the top, how many bushels of oats will it hold?

8. How many cubic feet of stone in a pyramid 24 ft. square and 54 ft. high?

9. The gas tank of a company is 60 ft. in diameter, and when full of gas stands 50 ft. high. How many cubic feet of gas has been used when the tank stands 5 ft. high?

10. Each side of an octagonal room is 12 ft., the distance from the center to each corner is 16 ft., and the room is 15 ft. high. How many cubic feet of air does it contain?

11. A well is 34 ft. deep and 5 ft. in diameter. The water stands 10 ft. from the top. How many standard barrels does it contain?

12. A farmer in burying his potatoes makes 3 cone-shaped piles 3 ft. high and 12 ft. in circumference at the base. How many bushels has he?

IX

MEASURES OF TIME

95. Unit. The unit by which time is measured is the *solar day*. It is the amount of time required for the earth to make one complete revolution upon its axis. It is computed from the instant the sun is at the highest point on any stated meridian until it again shines at the highest point on that meridian. When the sun is at the highest point, it is noon, and the time is known as 12 M., or *meridian*. If the sun has passed the highest point, the time is afternoon, and it is marked P.M., or *post-meridian* (after meridian); if this time has not yet arrived, the time is forenoon, and it is marked A.M., or *ante-meridian* (before meridian).

The day as used in practice begins when the sun is on the opposite side of the earth, or to midnight, and lasts until the succeeding midnight.

The *solar year* is the time required for the earth to make one complete revolution around the sun. It is 365 da. 5 hr. 48 min. 49.7 sec. long. This being nearly 365$\frac{1}{4}$ da., the length of the year is 365 da., with 1 da. added in each 4 yr., which day is counted as the 29th day of February. Since in 4 yr. the 5 hr. 48 min. and 49.7 sec. each year will not amount to one full day, the extra day is omitted from each centennial year not divisible by 400. Thus, the year 1900 was not a leap year, but the year 2000 will be.

NOTE. — In most business transactions 30 days are considered 1 month, and 12 months a year. The calendar length of the months varies. January, March, May, July, August, October, and December have 31 days; April, June, September, and November have 30 days; while February has 28 days in all years except leap years, when it has 29 days.

TABLE

There are:	60 seconds (sec.)	in 1 minute	(min.)
	60 minutes	in 1 hour	(hr.)
	24 hours	in 1 day	(da.)
	7 days	in 1 week	(wk.)
	30 days	in 1 month	(mo.)
	12 months	in 1 year	(yr.)
	365 days	in 1 year	
	366 days	in 1 leap year	

96. Difference in Time. There are two methods of finding the difference in time between certain dates. The *exact* method takes into account the calendar months intervening. It is used in determining the date of maturity of notes or other legal paper when the time period is given in days, and also by many banks in calculating discount. The *commercial* method counts the year as made up of 12 months of 30 days each. In this method the difference in time is usually found by what is termed *compound subtraction.*

97. Compound Subtraction in Time. If we desire to find the difference in time between May 24, 1918, and March 16, 1922, we write the dates as above, putting months under months, days under days, etc. As 24 days cannot be taken from 16 days, 1 month must be reduced to days, making 46 days in all. There would, then, be a difference of 22 days. One month being used, there are but 2 months remaining in the minuend, and 1 year or 12 months must be added, making 14 months. Subtracting 5 months, the remainder is 9 months. One year of 1922 being used, the date becomes 1921; and subtracting 1918, we have 3 years. The whole difference in time, then, is 3 years, 9 months, 22 days.

1922	3 mo.	16 da.
1918	5 mo.	24 da.
3 yr.	9 mo.	22 da.

LONGITUDE AND TIME

98. Measurement of Circles. For measuring angles, a unit called a *degree* is used. It is $\frac{1}{90}$ of a right angle. The entire angular measure around the center of a circle is equivalent to four right angles, or 360 degrees.

The part of a circumference opposite each degree angle, or $\frac{1}{360}$ of a circumference, is also called a *degree* (written °). In the same or equal circles, one degree of arc measures one degree of angle. If the circle is larger, the length of the arc increases, while the angle remains the same. The degree of arc means, simply, $\frac{1}{360}$ of a circumference. The derived units are shown in the table given below.

TABLE

There are:	60 seconds (″)	in 1 minute (′)
	60 minutes	in 1 degree (°)
	360 degrees	in 1 circle

99. Latitude and Longitude. When the units of circular measure are applied to the measurement of the earth's circumference, they are of use for locating places on the globe and for measuring distances.

For fixing locations north and south, great imaginary circles termed *parallels* are drawn one degree apart and parallel with the equator. The number of degrees north or south of the equator determines the *latitude.* These circles being parallel, the distance between them, or the length of a degree of latitude, is always nearly the same. Owing, however, to the curvature of the earth, a degree of latitude varies slightly, it being 68.72 miles at the equator, and 69.34 miles at the poles. Sixty-nine and sixteen hundredths miles has been adopted as the standard length of a degree of latitude. $\frac{1}{60}$ of a degree of latitude is sometimes known as a *geographical*

mile, to distinguish it from the ordinary or *statute* mile. The geographical mile is, therefore, equal to 1.152⅔ statute miles.

For measuring east and west, great circles, called *meridians*, pass through the poles. The distance between them varies from nothing at the poles, where they meet, to $\frac{1}{360}$ of the circumference of the earth at the equator. A degree of longitude varies in length, then, from 69.16 miles at the equator to nothing at the poles.

Distance east or west is indicated by the number of degrees or meridians east or west of a given meridian designated as a *principal meridian* for such purpose. The meridians of Greenwich, England, and Washington, D.C., are both used as principal meridians.

100. Difference in Longitude. If two places are both east or both west of the principal meridian, the difference in their longitude is found by subtracting the longitude of one from that of the other. Thus, the longitude of New York is 74° 0′ 3″ W. (Greenwich), and that of Chicago is 87° 36′ 42″ W. To find the difference, write the less number under the greater, putting degrees under degrees, minutes under minutes, and seconds under seconds (*a*). When the minuend numbers are larger, ordinary subtraction will give the result. In this case it is 13° 36′ 39″. Should some of the minuend numbers be smaller, the same method as that used in finding the difference of time (Sec. 97) should be used. Thus, in finding the difference in longitude between New

(*a*)

87°	36′	42″	W.
74°	0′	3″	W.
13°	36′	39″	

(*b*)

84°	26′	0″
74°	0′	3″
10°	25′	57″

(*c*)

74°	0′	3″	W.
2°	20′	15″	E.
76°	20′	18″	

York and Cincinnati (84° 26′ 0″ W.) (*b*), it is necessary to change one minute to seconds before the subtraction of 3″ can be made.

(*d*)

87°	36′	42″	W.
13°	23′	43″	E.
101°	0′	25″	

If the two places are one east and one west of the principal meridian, the difference in their longitude is found by adding them. Thus, the difference in longitude between New York and Paris (2° 20′ 15″ E.) is found, by adding, to be 76° 20′ 18″ (*c*), and between Chicago and Berlin (13° 23′ 43″ E.) is found to be 101° 0′ 25″ (*d*).

101. Relative Time. Difference in longitude is often expressed by the difference in the time of two given places. When the time of two places is given at the same instant, their difference in time may be found by compound subtraction. If the times of two places are both A.M. or both P.M., the difference in their time is found by subtracting (*a*).

(*a*)

New York	4 hr.	55 min.	53	sec. P.M.
Boston	4 hr.	44 min.	15	sec. P.M.
		11 min.	38	sec.

When the times given are one A.M. and one P.M., the difference in their time is found by adding 12 hr. to the P.M. time and subtracting the A.M. time from the sum (*b*).

(*b*)

Berlin	1 hr.	20 min.	42	sec. P.M.
Chicago	6 hr.	36 min.	$40\frac{1}{3}$	sec. A.M.
	6 hr.	44 min.	$1\frac{2}{3}$	sec.

102. Comparative Tables. As the earth revolves through 360° of longitude in 24 hours, it revolves through 1° in $\frac{1}{360}$ of 24 hours, or 4 minutes. In this way the following tables are derived.

TABLE 1

There are: 360° longitude in 24 hours of time
1° longitude in 4 minutes of time
1′ longitude in 4 seconds of time
1″ longitude in $\frac{1}{15}$ seconds of time

TABLE 2

There are: 24 hours for 360°
1 hour for 15°
1 minute for 15′
1 second for 15″

103. Reduction. Inasmuch as the time of day depends upon the position of the sun in the heavens, a place west of another will have an earlier time, and a place east will have a later time. A difference of longitude, then, will make a difference in time, and if we know the difference in longitude between two places, we may calculate their difference in time. Thus, the difference in longitude between New York and Paris is 76° 20′ 18″. Since 1° in longitude makes a difference of 4 minutes in time, 76° would make a difference of 304 minutes (4 min. × 76), or 5 hr. 4 min. 1′ making 4 sec. difference, 20.3′ (20′ 18″) would make 81.2 sec., or 1 min. 21.2 sec. All together, there would be a difference in time of 5 hr. 5 min. 21.2 sec. The problem may be written in the manner shown, and solved by the method of compound multiplication, which combines multiplication of each part and a reduction to higher denominations.

	76	20.3
		4
(*a*)	304	81.2
(*b*)	5 hr. 5 min. 21.2 sec.	

The first product (*a*) is that from simple multiplication, and the second (*b*) is the result of the reduction. The seconds of longitude should be reduced to decimals of a minute before multiplication.

On the other hand, when the difference in time between two places is known, the difference in longitude may be found by dividing such time by 4, using the method of compound division. Thus, if the difference in time between New York and Paris were known to be 5 hr. 5 min. 21.2 sec. (305 min. 21.2 sec.), the longitude difference would be found by taking $\frac{1}{4}$ of the time (expressed in minutes and seconds). This would be 76° 20.3′ (76° 20′ 18″).

$$\begin{array}{r|ll} 4 & 305 \text{ min.} & 21.2 \text{ sec.} \\ \hline & 76^\circ & 20.3' \end{array}$$

The minute remaining in the first division is changed to seconds and added to the second dividend before again dividing by four.

104. Standard Time. If 15° of longitude make a difference of one hour in time (Sec. 102), it is plain that as one traveled west the time, as shown by his watch, would grow faster than the local (meridian) time along the route. This varying time led to many inconveniences, particularly in railroad management.

In 1883, a system of *standard* time was adopted, and it is now in general use. By this plan, the United States is divided into four time belts, each about 15° in width.

Over the whole of each belt exactly the same time is used. It is the local or sun time at each standard meridian. These meridians are the 75th, which passes near Philadelphia; the 90th, near St. Louis; the 105th, near Virginia City, Mont.; and the 120th, near Carson City, Nev. The belts are known respectively as Eastern, Central, Mountain, and Pacific, and extend about $7\frac{1}{2}$° on either side of the meridian. The standard meridians, being 15° apart, the difference in time between them and their belts is exactly one hour. Hence, in traveling westward, watches should be set back one hour on crossing the dividing line between these belts, and in traveling eastward set forward one hour.

The dividing line between the belts is quite irregular, owing to the necessity of railroads avoiding a change of time at any but division stations. Similar devices for using

STANDARD TIME IN THE UNITED STATES

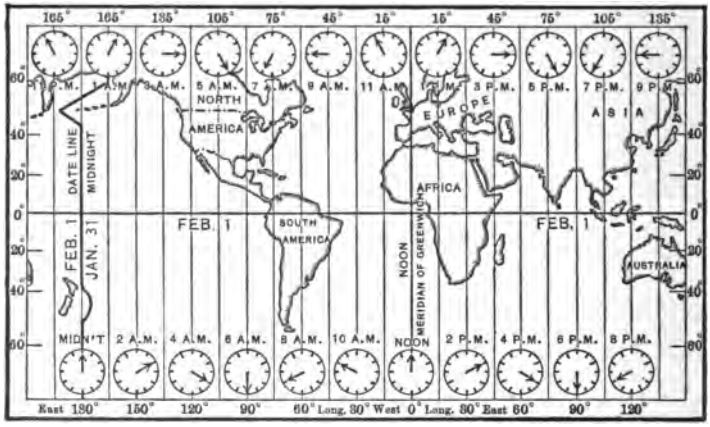
exactly the same time over given areas are now in use in the countries of Europe and in the European colonies of Asiatic and East Indian countries.

105. International Date Line. In traveling around the earth east or west, the fact that the earth is rotating from west to east has an effect upon the apparent time of day. In traveling east, the earth travels with one, but the sun seems to travel in an opposite direction, and one therefore shortens his day. In traveling around the earth to the east, one's time would be so shortened that it would take a day more for the journey than the actual time consumed would make if divided into days of twenty-four hours each.

On the other hand, if one traveled west around the world, it would seem to take a day less than the actual time consumed.

In navigation, this phenomenon caused annoyance, for ships going in opposite directions would often be a day apart in

their dates. To avoid this, a line called "The International Date Line" has been established. This line follows, approximately, the 180th meridian from Greenwich, diverging from such meridian whenever necessary to avoid land, because of the confusion of dates that would result through the use of different dates on the same body of land. The fact that the 180th meridian east or west of Greenwich passes between



INTERNATIONAL DATE LINE

North America and Asia, and traverses the ocean save for a few small islands, makes it a convenient longitude for such a purpose. Whenever ships pass this line going eastward, they subtract a day from their reckoning, and when going westward, they add a day.

106. Longitude of Leading Cities. The longitude of the cities given below may be determined from their difference in time from the time of Greenwich, which is given with the direction indicated.

	H.	M.	S.			H.	M.	S.	
Albany	4	55	6.8	W.	Annapolis, Md.	5	5	56.5	W.
Ann Arbor, Mich.	5	34	55.2	W.	Baltimore, Md.	5	6	26.0	W.

	H.	M.	S.			H.	M.	S.	
Berlin	0	53	34.9	E.	Hong Kong	7	36	41.9	E.
Bombay	4	51	15.7	E.	Liverpool	0	12	17.3	W.
Bordeaux, France	0	2	5.4	W.	Madrid, Spain	0	14	45.4	W.
Brussels, Bel.	0	17	28.6	E.	Melbourne, Vic.	9	39	54.1	E.
Calcutta	5	53	20.7	E.	Moscow	2	30	17.2	E.
Cambridge, Eng.	0	0	22.7	E.	New Orleans	6	0	13.9	W.
Canton, China	7	33	46.3	E.	New York	4	55	54.6	W.
Cape Good Hope	1	13	58.0	E.	Paris	0	9	20.9	E.
Chicago	5	50	26.7	W.	Philadelphia	5	0	38.5	W.
Cincinnati	5	37	41.3	W.	Rome	0	49	55.6	E.
Denver, Col.	6	59	47.6	W.	San Francisco	9	9	42.8	W.
Dublin, Ireland	0	25	21.1	W.	Seattle	8	9	19.9	W.
Edinburgh	0	12	43.1	W.	St. Louis	6	0	49.1	W.
Glasgow	0	17	10.6	W.	Providence, R.I.	4	45	37.5	W.

PROBLEMS

1. A note dated May 15 was paid July 6. How many days did it run?

SOLUTION:

16 days in May.
30 days in June.
6 days in July.
52 days from May 16 to July 6.

NOTE. — The day on which the note is given is not counted, but the day on which it is due is counted.

Find exact number of days from:

2. Aug. 7 to Oct. 28.

3. Dec. 14 to Feb. 12.

4. May 20 to July 25.

5. Aug. 18 to Nov. 14.

6. Sept. 28 to Jan. 31.

7. March 13 to Nov. 26.

8. May 3 to Dec. 5.

9. Jan. 1 to March 13.

10. Sept. 15 to Dec. 31.

By compound subtraction find difference in time between:

1. Nov. 18, 1903, July 7, 1906.

2. May 17, 1899, Sept. 16, 1903.

3. Feb. 11, 1910, Jan. 17, 1903.

4. Oct. 17, 1903, Dec. 6, 1907.

5. Aug. 26, 1902, Feb. 15, 1908.

6. May 25, 1901, Aug. 3, 1904.

REDUCTION:

1. The difference in time between two places is 2 hr. 15 min. What is the difference in longitude?

2. When it is noon at Chicago, what time is it 15° 15′ west of Chicago? 32° 15′ 30″ east?

3. A man travels from Cincinnati until his watch is 45 minutes fast. In what direction and through how many degrees has he traveled?

4. What is the difference in time between two places whose longitudes are 80° W. and 65° W.? Two places 16° 18′ E. and 90° 0′ 12″ W.?

5. By use of the preceding table, find the longitude of Albany, Baltimore, Berlin, Calcutta, Dublin, Madrid, Paris, Rome, and Providence.

6. When it is 11 A.M. at Washington, it is 10 hr. 7 min. 4 sec. at St. Louis. What longitude is St. Louis west of Washington?

7. Determine the time and date at Washington, D.C., Hongkong, Cape Good Hope, and Moscow when it is midnight, Jan. 1, at Greenwich.

8. When it is noon, Feb. 15, at Glasgow, what time and date is it at Denver? At Melbourne? At New York? At Brussels?

9. When it is 8 P.M. at Philadelphia, what is the time at Liverpool? At Paris? At Berlin? At Rome?

X

MEASURES OF WEIGHT

107. Units of Weight. Weight is the downward pressure of bodies toward the earth, or the measure of the attraction of gravitation. The English Troy *pound* has been adopted as the unit. As fixed by the English law, this is the weight of 22.7944 cu. in. of pure water at its greatest density. The use of this unit and its Troy subdivisions is restricted to weighing gold, silver, jewels, etc. The Troy pound is divided into twelfths, known as ounces, which in turn are divided into 20 pennyweights, each composed of 24 grains. There are, then, 5760 grains in a pound Troy.

Troy Weight Table

There are:

24 grains (gr.) in 1 pennyweight (pwt.)
20 pennyweights in 1 ounce (oz.)
12 ounces in 1 pound (lb.)

Comparisons

1 lb. = 12 oz. = 240 pwt. = 5760 gr.
1 oz. = 20 pwt. = 480 gr.
1 pwt. = 24 gr.

108. Apothecaries' Weight. For weighing drugs, in compounding prescriptions, etc., the Troy ounce is differently divided. The table of Apothecaries' weight follows.

TABLE

There are:	20 grains (gr.)	in 1 scruple (sc.)
	3 scruples	in 1 dram (dr.)
	8 drams	in 1 ounce (oz.)
	12 ounces	in 1 pound (lb.)

109. Avoirdupois Weight. For weighing ordinary bulky articles, such as coal, grain, groceries, etc., the Avoirdupois pound is used. It consists of 7000 Troy grains, and has been fixed by Congress (1901) as the weight of 27.7015 cu. in. of distilled water, at 62° Fahrenheit, weighed with brass weights in air, with the barometer at 30 in.

TABLE

There are:	16 ounces (oz.)	in 1 pound (lb.)
	100 pounds	in 1 hundredweight (cwt.)
	20 hundredweight	in 1 ton (T.)

COMPARISONS

1 ton = 2000 lb.
1 lb. = 7000 grains
1 oz. = 437.5 grains

NOTE. — At the Custom House, and to some extent in mining, the *long ton* of 2240 pounds is still used. As a subdivision, 112 pounds are called a hundredweight.

110. Bushel Weights. In handling grains and many other products for shipment, the dry-measure units are seldom used. It is much more convenient to weigh in bulk and allow a certain weight per bushel. Many states have, by statute, fixed the number of pounds per bushel of various products. The result of this practice has been a lack of uniformity among the states. In 1901 Congress fixed the minimum weight per bushel of certain articles of produce as:

	Pounds		Pounds
Wheat	60	Dried apples	26
Corn, ear	70	Dried peaches	33
Corn, shelled	56	Clover seed	60
Buckwheat	48	Flax seed	56
Barley	48	Millet seed	50
Oats	32	Hungarian	50
Peas	60	Timothy seed	45
White beans	60	Blue grass	44
Castor beans	46	Hemp seed	44
Rye	56	Corn meal	48
White potatoes	60	Ground peas	24
Sweet potatoes	55	Bran	20
Onions	57	Malt	34
Turnips	55		

Note. — In many places the weight per bushel has been discarded, grain and other materials being bought by hundredweight. This plan has many advantages.

Salt. The weight per bushel of salt as adopted by different states ranges from 50 to 80 pounds. Coarse salt in Pennsylvania is reckoned at 70 pounds, and in Illinois at 50 pounds per bushel. Fine salt in Pennsylvania is reckoned at 62 pounds, in Kentucky and Illinois at 55 pounds per bushel.

PROBLEMS

1. How many pounds in 1 T. 6 cwt.? In 4 cwt.? In 6 T. 8 cwt.?

2. How many ounces in 5 lb.? In 8 lb.? In 10 lb.? In 16 lb.? (Troy.)

3. How many grains in 3 pwt.? In 5 pwt.? In 8 pwt.? In 12 pwt.? In 16 pwt.?

4. How many tons in 40 cwt.? In 50 cwt.? In 60 cwt.? In 80 cwt.? In 100 cwt.?

5. How many ounces Troy in 4 lb.? In 8 lb.? In 20 lb.? In 15 lb.?

6. How many grains in 3 oz. 16 pwt. 12 gr.? In 4 oz. 6 pwt. 20 gr.?

7. How many bushels of ear corn in a load weighing 2650 lb.?

8. What will 2680 lb. shelled corn cost at 60¢ per bushel?

SOLUTION:

$$\begin{array}{r} .15 \quad 1430 \\ \$.\not{6}\not{0} \times \not{2}\not{8}\not{6}\not{0} = \$30.64 \\ \hline \not{5}\not{8} \\ \not{1}\not{4} \\ 7 \end{array}$$

9. What will 3670 lb. wheat cost at 78¢ per bushel?

10. What will 2750 lb. potatoes cost at 45¢ per bushel?

11. If a man weighs 156 lb. Avoirdupois, what will he weigh in Troy weight?

12. A farmer raises 7600 lb. rye, 1856 lb. clover seed, 1375 lb. flax, 4532 lb. onions, 458 lb. timothy seed. How many bushels of each did he raise?

13. A cubic foot of water weighs 62½ lb. At that rate, what will be the weight of the water in a tank 4 × 6 × 15 ft.?

14. A grocer's boy sold 10 lb. coffee by mistake, weighing it on a druggist's scales. How much did the grocer lose or gain, if the coffee was worth 20¢ a pound?

15. A farmer sold 27,650 lb. corn at 65¢ per bushel, 48,500 lb. wheat at 82¢ per bushel, 15,810 lb. oats at 35¢ per bushel. Find the total selling price.

16. A horse requires 12 lb. of oats and 20 lb. of hay per day. What will it cost to feed him a year, if oats are worth 26¢ a bushel, and hay $9.75 a ton?

17. A dealer purchased in a day 26,040 lb. oats at 28¢ a bushel, 19,654 lb. wheat at 76¢ per bushel, 28,642 lb. corn at 49¢ per bushel, and 15,060 lb. barley at 98¢ per bushel. The oats are sold at an advance of 4¢, the wheat at 5¢, the corn at 7¢, and the barley at 6¢. Find the gain.

18. A farmer raised on his farm 28,540 lb. wheat, 27,486 lb. rye, 8752 lb. potatoes, 54,660 lb. hay, 40,500 lb. oats, and 600 lb. beans. He sold the wheat for 65¢, the rye for 57¢, the potatoes for $1.25, the beans for $3.50, the oats for 32¢ per bushel, and the hay for $8.75 per ton. Find total returns. Find his gain, if he paid $375 for labor, $245 for seed, $175 for machinery, and $60 for hauling the crops to market.

19. A coal dealer bought 326 tons of coal by the long ton at $3.75 per ton, and sold it by the short ton at $6.50. How much did he gain?

XI

MEASURES OF VALUE

111. The Unit of Value. Congress has designated the *dollar* as the unit of value for the United States, to be composed of 23.22 grains of pure gold. For greater hardness, the pure metal is mixed with copper or an alloy of copper and silver, nine parts gold and one of alloy, making the full weight of a gold dollar 25.8 grains. The silver dollar contains 371.25 grains of pure silver; 41.25 grains of copper are added for hardness.

From the standard unit, $1, other units are derived, using decimal divisions and multiples; thus, $10 is called an eagle, $\frac{1}{10}$ of $1 is called a dime (French *disme* or ten) or ten cents; $\frac{1}{100}$ of a dollar is 1 cent (Latin *centum* or a hundred), and $\frac{1}{1000}$ of a dollar is 1 mill (Latin *mille* or a thousand). While theoretically all these divisions are made, practically we use but dollars and cents in writing or reading expressions of value.

112. Coins. For the convenience of business, Congress has caused to be minted the following coins:

Gold. Twenty dollars, sometimes called the double eagle, ten dollars or eagle, and five dollars or half eagle.

Silver. One dollar, half dollar or fifty cents, quarter or twenty-five cents, and dime. All except the first are *subsidiary.*

Minor Coins. Nickel or five cents, and one-cent piece.

NOTE. — The coinage of the gold dollar was discontinued pursuant to an act of Congress, Sept. 26, 1890.

113. Paper Money. For larger denominations, and for convenience in handling larger amounts of money, Congress

has authorized the issue of five different kinds of paper money, printed in various denominations. These are United States Notes or Greenbacks, Silver Certificates, Gold Certificates, Treasury Notes of 1890, and National Bank Notes.

114. Legal Tender. To facilitate trade and render it more secure, governments declare certain kinds of money to be "legal tender," *i.e.* money that must be accepted by a creditor in payment of a debt.

The following kinds of money are full legal tender: all gold coins, silver dollars, Treasury notes of 1890, and United States notes (except for duties on imports and interest on the public debt). National Bank notes and gold and silver certificates are not legal tender, but, because they are redeemable at the United States Treasury in legal tender money, they pass current without question. Subsidiary silver coins are legal tender for ten dollars or less, and minor coins for 25¢ or less.

115. Sending Money and Valuables. For a fee of 10 cents a letter or parcel containing money or other articles of value may be sent by registered mail. Registered mail is delivered only to the addressee, or on his written order, and, if not known to the authorities, such addressee must be identified. A receipt is taken upon delivery, and is returned to the sender of the letter or parcel. This receipt is, under the law, *prima facie* evidence of delivery. In case of loss, the government indemnifies the sender for a value up to $50.

By Express. Express companies will transport money under guarantee of its safe delivery, and will indemnify losses in full. The cost of such service, however, and its inconvenience, render it little used for ordinary business transactions.

Exchange. By exchange is meant a system of making money payments without the actual transportation of money. It is accomplished through the exchange of credits. The principal systems of exchange are given below.

116. Post Office Money Orders. The government provides a simple and safe means of exchange through the postal system. Offices designated as "money-order offices" are empowered to issue orders on other similarly designated offices for an amount not exceeding $100 to any one order. For sums above $100 more than one order must be purchased.

To obtain a money order, a prescribed form of application blank must be filled out. The rates charged, in addition to the face value of the order, are printed on the back of the order blank, and the cost of an order for a given amount may be easily computed. The government is liable for payment to the wrong person, provided the wrong payment was not brought about through the fault of the remitter, payee, or indorsee. Money orders may be obtained or cashed at over 35,000 offices in the United States.

117. Express Money Orders. Express companies issue money orders in much the same form and at practically the same rates as the Post Office Department. A receipt is given the sender, and, in case the order is lost, a refund may be had upon filing an indemnity bond guaranteeing the company against loss should the lost order be cashed.

The limit of each order is $50, but any number may be purchased. When indorsed, these orders are usually accepted by banks at their face value.

118. Bank Exchange. Banking houses maintain deposits or accounts with other banks, called "correspondents." They sell orders, called *drafts*, on these correspondents, charging a small fee therefor. In all ordinary business and for relatively small sums, the fee is a fixed charge. Ten cents for small amounts, and fifteen cents per hundred dollars, are the rates charged in many banks. For larger amounts, and between large business houses, a certain fraction (per cent) of the amount of the draft issued is sometimes charged.

NOTE. — As this phase of exchange deals with per cents and banking procedure, it is reserved for further discussion in the chapter on "Banking and Discount."

Some banks issue drafts payable at any one of a number of other banks. Such drafts are known as *bank money orders*, and are usually charged for at the same rate as post office or express money orders.

119. Exchange by Wire. Telegraph companies, for a fixed fee in addition to the cost of sending the necessary telegrams, instruct payees by wire to call at a designated telegraph office to receive a certain sum of money, and, in the same way, direct the manager of such office to pay the amount to a particular person on demand.

Express companies also order money paid by telegram, instructing their agent at a certain place to deliver to the payee a given amount.

Telephone companies direct their managers at other places to pay given amounts. Banks, too, will order the payment of money by telegraph.

Exchange by wire is necessarily more expensive, as the fees are quite large. Such a method is intended for use only in emergencies.

120. Foreign Exchange.

FOREIGN COINS

English Money.	There are:	4 farthings in 1 penny (*d.*) 12 pence in 1 shilling (*s.*) 20 shilling in 1 pound (£)
French Money.	There are:	10 centimes in 1 decime 10 decimes in 1 franc
German Money.	There are:	100 pfennigs in 1 mark
Spanish Money.	There are:	100 céntesimos in 1 peseta

NOTE.—France, Belgium, Greece, Italy, and Switzerland constitute what is known as the "Latin Monetary Union," and their coins are

Value of Foreign Coins in United States Money

Country	Standard	Monetary Unit		Value in Terms of U.S. Gold Dollar	Coins
Argentine Republic	Gold	Peso		$0.965	Gold: argentine ($4.824) and ½ argentine. Silver: peso and divisions.
Austria-Hungary	Gold	Crown		.203	Gold: 10 and 20 crowns. Silver: 1 and 5 crowns.
Belgium	Gold	Franc		.193	Gold: 10 and 20 francs. Silver: 5 francs.
Bolivia	Gold	Boliviano		.389	Gold: *Silver boliviano and divisions.
Brazil	Gold	Milreis		.546	Gold: 5, 10, and 20 milreis. Silver: ½, 1, and 2 milreis.
British Possessions, N. A. (except Newfoundland)	Gold	Dollar		1.000	
Central Amer. States —					
Costa Rica	Gold	Colon		.465	Gold: 2, 5, 10, and 20 colons ($9.307). Silver: 5, 10, 25, and 50 centimos.
British Honduras	Gold	Dollar		1.000	
Guatemala, Honduras, Nicaragua, Salvador	Silver	Peso		.375	Silver: peso and divisions.
Chile	Gold	Peso		.365	Gold: escudo ($1.825), doubloon ($8.650), and condor ($7.300). Silver: peso and divisions.
China	Silver	Tael	Amoy	.614	
			Canton	.612	
			Cheefoo	.587	
			Chin Kiang	.600	
			Fuchau	.568	
			Haikwan (Customs)	.625	
			Hankow	.575	
			Kiaochow	.595	
			Nankin	.606	
			Niuchwang	.576	
			Ningpo	.590	
			Peking	.599	
			Shanghai	.561	
			Swatow	.567	
			Takau	.618	
			Tientsin	.595	
		Dollar	Hongkong	.404	
			British	.404	
			Mexican	.407	

Colombia	Gold	Dollar	1.000	Gold: condor ($9.647) and double condor. Silver: peso.
Denmark	Gold	Crown	.268	Gold: 10 and 20 crowns.
Ecuador	Gold	Sucre	.487	Gold: 10 sucres ($4.8665). Silver: sucre and divisions.
Egypt	Gold	Pound (100 piasters).	4.943	Gold: pound (100 piasters), 5, 10, 20, and 50 piasters. Silver: 1, 2, 5, 10, and 20 piasters.
Finland	Gold	Mark	.193	Gold: 20 marks ($3.859), 10 marks ($1.93).
France	Gold	Franc	.193	Gold: 5, 10, 20, 50 and 100 francs. Silver: 5 francs.
German Empire	Gold	Mark	.238	Gold: 5, 10, and 20 marks.
Great Britain	Gold	Pound sterling	4.866½	Gold: sovereign (pound sterling) and ½ sovereign.
Greece	Gold	Drachma	.193	Gold: 5, 10, 20, 50, and 100 drachmas. Silver: 5 drachmas.
Haiti	Gold	Gourde	.965	Gold: 1, 2, 5, and 10 gourdes. Silver: gourde and divisions.
India [British]	Gold	Pound sterling †.	4.866½	Gold: sovereign (pound sterling). Silver: rupee and divisions.
Italy	Gold	Lira	1.93	Gold: 5, 10, 20, 50, and 100 lire. Silver: 5 lire.
Japan	Gold	Yen	.498	Gold: 5, 10, and 20 yen. Silver: 10, 20, and 50 sen.
Liberia	Gold	Dollar	1.000	
Mexico	Gold	Peso ‡	.498	Gold: 5 and 10 pesos. Silver: dollar § (or peso) and divisions.
Netherlands	Gold	Florin	.402	Gold: 10 florins. Silver: 2½, 1 florin and divisions.
Newfoundland	Gold	Dollar	1.014	Gold: 2 dollars ($2.028).
Norway	Gold	Crown	.268	Gold: 10 and 20 crowns.
Panama	Gold	Balboa	1.000	Gold: 1, 2½, 5, 10, and 20 balboas. Silver: peso and divisions.
Persia	Silver	Kran	.069	Gold: ½, 1, and 2 tomans ($8.409.) Silver ¼, ½, 1, 2, and 5 krans.
Peru	Gold	Libra	4.866½	Gold: ½ and 1 libra. Silver: sol and divisions.
Philippine Islands	Gold	Peso	.500	Silver peso: 10, 20, and 50 centavos.
Portugal	Gold	Milreis	1.080	Gold: 1, 2, 5, and 10 milreis.
Russia	Gold	Ruble	.515	Gold: 5, 7½, 10, and 15 rubles. Silver: 5, 10, 15, 20, 25, 50, and 100 copecks.
Spain	Gold	Peseta	.193	Gold: 25 pesetas. Silver: 5 pesetas.
Straits Settlements	Gold	Pound sterling ¶	4.866½	Gold: sovereign (pound sterling). Silver: dollar and divisions.
Sweden	Gold	Crown	.268	Gold: 10 and 20 crowns.
Switzerland	Gold	Franc	.193	Gold: 5, 10, 20, 50, and 100 francs. Silver: 5 francs,
Turkey	Gold	Piaster	.044	Gold: 25, 50, 100. 250, and 500 piasters.
Uruguay	Gold	Peso	1.034	Gold: peso. Silver: peso and divisions.
Venezuela	Gold	Bolivar	.193	Gold: 5, 10, 20, 50, and 100 bolivars. Silver: 5 bolivars.

NOTE. — The coins of silver-standard countries are valued by their pure silver contents, at the average market price of silver for the three months preceding the date at which a value is given by the Treasury Department at Washington.

* Gold standard adopted December 31, 1908, 12½ bolivianos equal the pound sterling or Peruvian pound ($4.866½).

† The sovereign is the standard coin of India, but the rupee ($0.3244½) is the current coin, valued at 15 to the sovereign.

‡ Seventy-five centigrams fine gold.

§ Value in Mexico, $0.498.

¶ The current coin of the Straits Settlements is the silver dollar issued on Government account and which has been given a tentative value of $0.567758⅓.

alike in weight and fineness, occasionally differing, however, in name. Most of the Central and South American states possess a standard coin.

121. Bills of Foreign Exchange. Drafts payable in foreign countries are either banker's or commercial bills of exchange. They are based upon credits in the foreign cities in the same way as are bank drafts upon cities in our own country, the latter being known, by way of distinction, as *domestic exchange*. Banker's bills were formerly drawn in duplicate or triplicate, each copy being sent by a different route for safety. Rapid and safe mail service renders this no longer necessary, particularly between Europe and America. Accordingly, foreign bills are now usually drawn singly, the same as domestic drafts.

All foreign bills are payable in the money of the country upon which they are drawn, and quotations of prices of foreign exchange give the cost in our money of one monetary unit of such country. Thus, an English pound sterling may be quoted as $4.86 or $4.88, etc.

The ebb and flow of commerce, with the balance of trade shifting from one country to another, causes changes in the value of credits in one country which may be owned in another, and these fluctuations are shown in daily quotations issued by banks and clearing houses. When there is a larger foreign credit than is needed for the demands of business, then foreign exchange is at a discount; when less than business requires, it is at a premium.

Certain banks in larger cities maintain reciprocal relations as correspondents with banks in certain foreign centers. Banks in smaller places often arrange to draw on such foreign correspondents through these larger banks, paying therefor as per quotations furnished.

122. Letters of Credit. Banks also issue *letters of credit* to persons who wish to travel abroad. They are addressed

to any of a list of correspondent banks in various cities in Europe, requesting them to furnish funds to the order of the holder up to a certain limit. Upon reaching one of the mentioned cities, a draft drawn by the holder of the letter for an amount within the limit will be cashed by the correspondent bank. The amounts drawn are indorsed on the letter by the banks cashing the drafts, and the letter itself accompanies the last draft which exhausts the limit of the credit. The advantage of these letters is in one being able to draw varying amounts, according to his needs, in a large number of places.

123. Foreign Money Orders. *Foreign money orders*, payable in the currency of the principal countries of the world, may be purchased of the United States government at any money-order office.

Some express companies issue *money orders* or *drafts* on important cities in foreign countries, and these drafts are payable in the money of such country at the company's agencies, or at certain banks with which they have arrangements as correspondents.

Travelers' checks, issued in certain denominations, with the foreign-money values in various countries printed on the face thereof, may be purchased in any number. These checks are payable at various banks and agencies, and pass current in almost any country for their face value in payment of ordinary expenses. A supply of these checks, in different denominations, forms a very convenient kind of exchange for travelers.

Some express companies also issue *letters of credit*, entitling the holder to draw money or travelers' checks thereon up to a certain limit. For such a letter of credit either cash must be deposited, or it must be secured either by a deposit of securities or the guarantee of a responsible bank, trust company, or banker.

Cable Transfers. Money may be paid by cable in the same way as by telegraph.

Shipments of gold coin or bullion are frequently made to restore a balance of credits in foreign exchange.

Miscellaneous Measures

124. Tradesman's Table.

There are: 12 units in 1 dozen (doz.)
12 dozen in 1 gross (gr.)
12 gross in 1 great gross

Twenty units are often called a score.

125. Stationer's Table.

Paper, particularly of the finer kinds, is commonly put up and sold according to the following units:

There are: 24 sheets in 1 quire
20 quires in 1 ream (480 sheets)
2 reams in 1 bundle
5 bundles in 1 bale

Paper houses are gradually adopting the commercial ream of 500 sheets, particularly for book and print papers. The larger units are disappearing from use.

PROBLEMS

1. The United States paid Spain $20,000,000 for the Philippines. What was the face of the bill, if the rate of exchange was 19.38 cents?

2. A merchant owes a bill of £300 8*s.* 10*d.* in London. What will it cost him for a bill, if the rate of exchange is $4.865?

3. A Boston importer owed a Dresden manufacturer 26,450 marks, and paid him by a bill of exchange. If the rate was 23.85 cents, what did it cost him?

4. I owe 4250 francs in Paris. If the rate is 19.38 cents, what will it cost me to pay the debt?

5. A merchant bought 4 cases musical instruments in Berlin, amounting to 3598.6 marks, and received a discount of $\frac{1}{10}$. If the exchange is worth 23.87 cents, what will the bill of exchange cost?

6. What will be the value of a sight draft on Berlin, amounting to 3598.6 marks, if the exchange is worth 23.88 cents?

7. What must be paid for a bill on Rome for 6750 lire, at 19¢?

XII

FRENCH METRICAL SYSTEM

127. The Metric System. The rapidly expanding use of the measures of the French Metrical System renders a knowledge of its units more and more a necessity in a business education. Its use is required in some of the departments of the government and is authorized in others. Government comparisons of standards are made by reference to the international metric units. The metric system is the legal standard in the Philippines and Porto Rico. So extensive is its use in foreign countries that manufacturers, particularly of scientific instruments and machinery, are putting out their products in metric sizes for the export trade.

The metric system is founded primarily upon the *meter*, which is the unit of length. All other units are so related to this as greatly to simplify calculations in which weights and measures are involved. It is this simple relation to one fundamental unit which makes it a system. Add to this the fact that it is a *decimal* system, and its extreme simplicity both in plan and ease of calculation is explained.

With a desire to found the unit upon something fixed, the French mathematicians took one ten-thousandth part of the earth's quadrant for a *Kilometer*, of which the more usable meter is a thousandth part. Under the direction of an International Bureau of Weights and Measures, established near Paris by the principal nations of the world, standard meters and kilograms were cast from platinum-iridium, and distributed to the various nations. By the law of 1893, all our common units are derived from these international standards, which are kept in the office of the National Bureau of Standards at Washington.

The plan of naming the derived units is equally simple. Derivatives from the Greek, viz. *deka* for ten, *hekta* for hundred, *kilo* for thousand, and *myria* for million, are prefixed to the name of the units for all multiples, while the fractional units are indicated by the Latin prefixes, *deci* for tenth, *centi* for hundredth, and *milli* for thousandth.

A practical knowledge of the metric system should be acquired by a study and use of the measures themselves. Each unit studied should be fixed by actual use, without regard to the nearest English unit, and the power to estimate accurately in its terms developed through drill. After such knowledge of the various units has been acquired, and one is able to think in these units, a study of comparative values between them and the English units has its value. Comparative tables are appended for reference only.

MEASURES OF LENGTH

128. The Unit of Length. As before stated, the *meter* is one thousandth part of a kilometer, which is one ten-thousandth part of the earth's quadrant. It is, therefore, one ten-millionth of the distance from the equator to the pole. From it all the other units of the French Metrical System are derived. The surveyor's chain is a dekameter or half dekameter in length.

TABLE

There are:

10 millimeters (mm.)	in 1 centimeter	(cm.)
10 centimeters	in 1 decimeter	(dm.)
10 decimeters	in 1 *meter*	(m.)
10 meters	in 1 dekameter	(Dm.)
10 dekameters	in 1 hektameter	(Hm.)
10 hektameters	in 1 kilometer	(Km.)

MEASURES OF AREA

129. Unit of Area. The unit of area is the *square meter.* In a like manner each denomination of linear measure is squared, forming the units of area. A square meter being 10 decimeters wide and 10 decimeters long, has 100 square decimeters. The same principle is applied to the other units.

TABLE

There are:		
100 sq. millimeters (sq. mm.)	in 1 sq. centimeter	(sq. cm.)
100 sq. centimeters	in 1 sq. decimeter	(sq. dm.)
100 sq. decimeters	in 1 *sq. meter*	(sq. m.)
100 sq. meters	in 1 sq. dekameter	(sq. Dm.)
100 sq. dekameters	in 1 sq. hektameter	(sq. Hm.)
100 sq. hektameters	in 1 sq. kilometer	(sq. Km.)

The chief use of the larger area units is in the measurement of land surfaces. For that purpose, the square dekameter is given a special name, and is known as the *are*, and the square hektameter as the *hectare*. The latter is the more commonly used land-area measure, while the square kilometer has no practical application.

MEASURES OF VOLUME

130. Units of Volume. The units of volume are derived in the same way, each linear unit being cubed. A cubic meter being 10 decimeters along each edge, contains 1000 cubic decimeters. Likewise, there are 1000 of each cubic denomination in the next higher one.

TABLE

There are:		
1000 cu. millimeters (cu. mm.)	in 1 cu. centimeter	(cu. cm.)
1000 cu. centimeters	in 1 cu. decimeter	(cu. dm.)
1000 cu. decimeters	in 1 *cu. meter*	(cu. m.)
1000 cu. meters	in 1 cu. dekameter	(cu. Dm.)

131. Capacity Units. For measuring capacity, the cubic decimeter is used. The special name of *liter* is given to it, from which are derived the denominations of capacity with the constant ratio of ten between successive units, and designated by the usual prefixes.

There are: TABLE

10 milliliters	(ml.)	in 1 centiliter	(cl.)
10 centiliters		in 1 deciliter	(dl.)
10 deciliters		in 1 *liter*	(l.)
10 liters		in 1 dekaliter	(Dl.)
10 dekaliters		in 1 hektaliter	(Hl.)

Wood Measure. For the measure of wood, the cubic meter is given a special name, the *stere*. This, likewise, may be treated decimally and a series of denominations formed.

MEASURES OF WEIGHT

132. Unit of Weight. For a unit of weight, one cubic centimeter of distilled water is taken, at its greatest density, in the latitude of Paris and at sea level. It is called a *gram*. Decimal denominations are derived from this in the same way as from the liter. A cubic decimeter or liter of pure water contains 1000 cubic centimeters, and hence weighs 1000 grams, or one kilogram. For very heavy articles the weight of a cubic meter of water is used, or 1000 kilograms, and this is called a *tonneau*.

There are: TABLE

10 milligrams	(mg.)	in 1 centigram	(cg.)
10 centigrams		in 1 decigram	(dg.)
10 decigrams		in 1 *gram*	(g.)
10 grams		in 1 dekagram	(Dg.)
10 dekagrams		in 1 hektagram	(Hg.)
10 hektagrams		in 1 kilogram	(Kg.)
100 kilograms		in 1 tonneau	(T.)

133. Decimal Methods. As the successive units of the metric system usually bear the ratio of ten to each other, various denominations are very conveniently written as a decimal of one denomination. Thus, 3 Dm. 5 m. 6 dm. 7 cm. 5 mm. would usually be written as 35.675 m.

For the same reason a change from one denomination to another may be accomplished by merely moving the decimal point. Thus, the above 35.675 m. may also be written as 3.5675 Dm., or 356.75 dm., etc. Obviously this ease of reduction is a decided advantage in the use of the metric system.

PROBLEMS

1. Write 5463 cm. as kilometers; as decimeters; as dekameters; as meters.

2. Write 5360 sq. m. as ares; as hektares.

3. Write 5200 cl. as liters; as hektaliters.

4. At 6¢ a meter, what will it cost to build a fence 54 Dm. 6 m. long?

5. How many rails 9 m. 4 dm. long will it take to build a railway 20 Km. 4 Hm. 2 Dm. long?

6. What is the weight of a liter of water in grams? What is the weight of a cubic meter of water? How many liters in it?

7. A circular lot is 27 meters in diameter. What is the area in ares?

8. How many ares in a rectangular field 8 Dm. long and 5 Dm. 4 m. 6 dm. wide?

9. How many cubic meters of water in a tank 10 m. long, 6 m. 7 dm. high, and 8 m. wide?

10. A fence is 5 boards high and 12 Hm. 2 Dm. long. How many boards 3 m. long are there in it?

11. How much carpet 1 m. wide will be needed to cover a room 6.4 m. long and 5.5 m. wide?

12. What will 50 l. of mercury weigh, if it is 13.5 times heavier than water?

13. If a pile of wood is 32 m. long, 5 m. 2 dm. wide, and 3 m. 6 dm. high, what is it worth at $2 a stere?

14. How much wheat will a bin 4 m. long, 3 m. 4 dm. wide, and 2 m. high hold? What is the value at 60¢ a liter? What is the weight?

15. How many tiles 40 cm. × 20 cm. will be used in tiling a floor 9 m. 6 dm. long and 5 m. 4 dm. wide?

16. If copper is 8.8 times as heavy as water, what is the weight of 8 cu. dm. of the metal?

17. What will it cost to plaster the walls and ceiling of a room that is 6 m. 5 dm. by 5 m. 8 dm. by 4 m. 2 dm., at 32¢ a square meter?

18. What will it cost to carpet a room 5 m. 4 dm. by 3 m. 2 dm. with carpet 8 dm. wide at 80¢ a meter?

19. What will it cost to paint the walls of a barn 15.5 m. long, 10 m. wide, and 9.5 m. high, at 35¢ per square meter?

20. Find the capacity of a tank 5 m. long, 3 m. 6 dm. wide, and 2 m. 6 dm. high.

21. What will be the cost of 4608 Kg. of hay at $12 a ton?

22. How many steres of wood in a pile 10 Dk. long, 3 m. 4 dm. wide, and 1.5 m. high?

23. Gold is 19.5 times as heavy as water. What is the weight of a cubic centimeter of gold?

24. How many jars, each containing 2.5 liters, can be filled from a cask containing 145.5 Dl.?

25. A room 6.3 m. long and 4 m. wide will require how many meters of carpet 8 dm. wide to cover it?

26. Find the area of the four walls of a room, 10.5 m. long, 6.5 m. wide, and 5.4 m. high. Area of ceiling? Cost to plaster the room at 35¢ a square meter?

27. What will it cost to paint the walls and ceiling of a room 8.5 m. × 5.4 m. × 4 m., deducting for four windows each 2 m. × 1 m., at 15¢ per square meter?

28. A room is 8 m. × 5 m. × 3.2 m. Deducting for five windows, each 2.1 m. × 1 m., 2 doors each 2.8 m. × 1.4 m., and a baseboard 2 dm. high, what will be the cost of plastering, at 45¢ a square meter? The cost of paper 4 dm. wide at $4.20 per roll of 10 meters? What is the cost of carpet 5.2 dm. wide, at $1.50 per meter?

COMPARATIVE TABLES

134. 1. Customary Units to Metric Units.

LENGTH	AREA
1 in. = 25.4001 mm.	1 sq. in. = 6.452 sq. cm.
1 ft. = .304801 m.	1 sq. ft. = 9.290 sq. dm.
1 yd. = .914402 m.	1 sq. yd. = .836 sq. m.
1 mi. = 1.60935 Km.	1 acre = .4047 Hm.

VOLUME	CAPACITY
1 cu. in. = 16.387 cu. cm.	1 fl. dr. = 3.70 cu. cm.
1 cu. ft. = .02832 cu. m.	1 fl. oz. = 29.57 mm.
1 cu. yd. = .765 cu. m.	1 qt. = .94636 l.
1 bushel = .35239 Hl.	1 gal. = 3.78543 l.

WEIGHT

1 gr. = 64.7989 mg.
1 av. oz. = 28.3495 g.
1 av. lb. = 45359 Kg.
1 troy oz. = 31.10348 g.

2. Metric Units to Customary Units.

LENGTH	SQUARE
1 m. = 39.3700 in.	1 sq. m. = 1550 sq. in.
1 m. = 3.28083 ft.	1 sq. m. = 10.764 sq. ft.
1 m. = 1.093611 yd.	1 sq. m. = 1.196 sq. yd.
1 Km. = .62137 mi.	1 Ha. = 2.471 A.

CUBIC	CAPACITY
1 cu. cm. = .0610 cu. in.	1 mm. = .27 fl. dr.
1 cu. dm. = 61.023 cu. in.	1 cl. = .338 fl. oz.
1 cu. m. = 35.314 cu. ft.	1 l. = 1.0557 qt.
1 cu. m. = 1.308 cu. yd.	1 Dl. = 2.6417 gal.
	1 Hl. = 2.8337 bu.

Weight

1 mg. = .01543 gr.
1 Kg. = 15432.36 gr.
1 Hg. = 3.5274 av. oz.
1 Kg. = 2.20462 av. lb.
1 T. = 2204.6 av. lb.

XIII

PERCENTAGE

135. Meaning and Use. By *per cent* is meant *hundredths*. The term is an abbreviation of the Latin *per centum*, or "by the hundred." As a further abbreviation, % is used.

Just as decimals are a development of certain forms of fractions having advantages in ease of operation, so percentage is the development of a certain kind of decimal fraction, having advantages for comparison in business transactions. A wide range of variation in the size of different parts of a whole may be indicated by hundredths, while operations are often made easy by frequent opportunities for using simple fractions. Since hundredths may be written as decimals, operations in percentage also possess all the advantages of decimals in ease and quickness of computation.

In Secs. 19 and 20 the methods of finding any number of hundredths were developed. In the treatment of simplified processes under Fractional Parts (Chapter IV), suggestions were made for shortening those methods. While not entering now upon the study of anything essentially new, a further and more systematic study of hundredths must be made that we may better understand percentage in its various applications to business.

136. Profit and Loss. Business enterprises are carried on in the expectation of making a *profit* for those who invest money or other wealth in them. At regular intervals, usually every year, a careful inventory is made of the results of the year's business, to find out how much profit has been made.

It sometimes happens that such an inventory shows that the business has been carried on at a *loss*. Whether a loss or a profit, the amount is first ascertained, and then, for purposes of comparison, the per cent that the loss or profit is of the capital invested is found.

Profit or loss is expressed, then, in terms of per cent. Because of the simplicity of the ideas involved, problems in profit and loss are used in the study of percentage processes.

137. Finding 50%, 25%, or 20%. Since there are one hundred hundredths in the whole of anything, *100% of anything is equal to the whole of it.* If 100% of anything is all of it, 50% is $\frac{1}{2}$ of it. Likewise, since $\frac{1}{4}$ of 100% equals 25%, then 25% of anything is $\frac{1}{4}$ of it; 20% of anything is $\frac{1}{5}$ of it, etc.

What per cent of anything is $\frac{3}{4}$ of it? What $\frac{2}{5}$? What $\frac{3}{5}$? What $\frac{4}{5}$?

SUMMARY

$100\% = \frac{100}{100}$	$25\% = \frac{1}{4}$	$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$
$50\% = \frac{1}{2}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$80\% = \frac{4}{5}$

ILLUSTRATIVE PROBLEMS

1. Mr. King invested \$400 in grain, and lost 50%. How much did he lose?

Since 50% of anything is $\frac{1}{2}$ of it, then his loss is $\frac{1}{2}$ of \$400, or \$200.

Stated thus:

$$50\% = \tfrac{1}{2}$$
$$\tfrac{1}{2} \text{ of } \$400 = \$200$$

2. A clerk spends \$30, which is 25% of his salary. What is his salary?

Since 25% of anything is $\frac{1}{4}$, then \$30 is $\frac{1}{4}$ of his salary, and $\frac{4}{4}$ are 4 times \$30, or \$120, his salary.

STATEMENT:

$$25\% = \tfrac{1}{4}$$
$$\tfrac{1}{4} \text{ of salary} = \$30$$
$$\$30 \times 4 = \$120, \text{ salary.}$$

3. A man having $75, spends $15 for a coat. What per cent of his money does he spend?

Since he has $75, and spends $15, he spends $\frac{15}{75}$, or $\frac{1}{5}$ of his money. But $\frac{1}{5}$ of anything is 20% of it; therefore he spends 20% of his money for the coat.

STATEMENT:

$\frac{15}{75} = \frac{1}{5}$

$\frac{1}{5} = 20\%$ spent for coat.

4. A man owns 300 A. of land, which is 50% more than he owned the year before. How much did he own the year before?

Since 50% of anything is $\frac{1}{2}$ of it, he would have $\frac{1}{2}$ more, or $1\frac{1}{2}$ times what he had the year before. But $1\frac{1}{2}$ times anything is $\frac{3}{2}$ of it; therefore 300 A. is $\frac{3}{2}$ of what he had, and $\frac{1}{2}$ is $\frac{1}{3}$ of 300 A., or 100 A. The whole of what he had, then, was twice 100 A., or 200 A.

STATEMENT:

50% = $\frac{1}{2}$.

$\frac{3}{2}$ of what he had = 300 A.

$\frac{1}{2} = \frac{1}{3}$ of 300 A., or 100 A.

$\frac{2}{2} = 100$ A. × 2, or 200 A., what he had the year before.

5. I sell a house for $600 and lose 20% by doing so. What did the house cost?

Since 20% of anything is $\frac{1}{5}$ of it, I have lost $\frac{1}{5}$, and have $\frac{4}{5}$ remaining. Then, $600 is $\frac{4}{5}$ of the cost, and $\frac{1}{5}$ is $\frac{1}{4}$ of $600, or $150. The whole cost, then, would be 5 times $150, or $750.

STATEMENT:

20% = $\frac{1}{5}$

$\frac{4}{5}$ of cost = $600

$\frac{1}{5} = \frac{1}{4}$ of $600, or $150

$\frac{5}{5} = \$150 \times 5$, or $750, cost.

PROBLEMS

1. A farmer invested $600 in hogs and lost 50%. How much did he lose?

2. A bookkeeper spends $28, which is 25% of his monthly salary. What is his salary?

3. A man owns 600 sheep, which is 50% more than he owned the year before. How many did he own the year before?

4. A horseman bought a horse for $160 and sold it for $200. What per cent did he gain?

5. A farmer had 150 hogs and lost 20% of them. How many did he have left?

6. An agent buys books for $3.25 and sells them for $6.50. What per cent did he gain?

7. A firm buys lots for $520 and sells them for $650. What per cent is the gain?

8. A man paid $80 for a horse and sold it for $120. What per cent did he gain?

9. A poultry man sold 240 chickens, which were $\frac{4}{5}$ of his flock. How many were there in the flock? What per cent of the flock did he sell?

10. Mr. Frank paid $80 for a horse. What per cent of his money did he pay if he had $200?

11. I sell a farm for $6000 and gain 25% by doing so. What did the farm cost me?

12. I sell a horse for $600 and lose 20% by doing so. What did the horse cost me?

13. A farmer lost 60 hogs by disease, which was 40% of his herd. How many had he at first?

14. A land agent had 1600 acres of land and sold 60% of it. How many acres did he sell?

15. A capitalist gave $5400, which was 75% of the amount he gained, to a public library. What did he gain?

16. A broker sold 30 shares of stock, which was 20% of all he had. How many shares did he have?

17. If A's money, $42, is 20% more than my money, how much have I?

18. If 84 sheep is 20% less than the number of sheep I have, how many have I?

19. A farmer buys goods for $60 and sells them for $48. What per cent does he lose?

138. Finding $33\frac{1}{3}\%$, $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, and $14\frac{2}{7}\%$.

$\frac{1}{3}$ of $100\% = 33\frac{1}{3}\%$
$\frac{1}{6}$ of $100\% = 16\frac{2}{3}\%$
$\frac{1}{8}$ of $100\% = 12\frac{1}{2}\%$
$\frac{1}{7}$ of $100\% = 14\frac{2}{7}\%$

Then,

$33\frac{1}{3}\% = \frac{1}{3}$	$14\frac{2}{7}\% = \frac{1}{7}$	$37\frac{1}{2}\% = \frac{3}{8}$
$16\frac{2}{3}\% = \frac{1}{6}$	$66\frac{2}{3}\% = \frac{2}{3}$	$62\frac{1}{2}\% = \frac{5}{8}$
$12\frac{1}{2}\% = \frac{1}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$	$87\frac{1}{2}\% = \frac{7}{8}$
		$28\frac{4}{7}\% = \frac{2}{7}$

It is well to remember that $\frac{2}{6} = \frac{1}{3}$, or $33\frac{1}{3}\%$; $\frac{3}{6} = \frac{1}{2}$, or 50%; $\frac{4}{6} = \frac{2}{3}$, or $66\frac{2}{3}\%$; $\frac{2}{8} = \frac{1}{4}$, or 25%; $\frac{4}{8} = \frac{1}{2}$, or 50%; and $\frac{6}{8} = \frac{3}{4}$, or 75%.

PROBLEMS

1. If \$15 is $33\frac{1}{3}\%$ of my money, how much have I?

2. A has \$72. B has $33\frac{1}{3}\%$ less. How much has B?

3. If A has \$48, which is $33\frac{1}{3}\%$ less than B's money, how much has B?

4. Mr. Hartman buys a buggy for \$63, which is $12\frac{1}{2}\%$ less than he paid for a horse. What did the horse cost?

5. C has 70 acres of land, which is $16\frac{2}{3}\%$ as much as B's. How much has B?

6. A drove 21 miles in a day, which was $16\frac{2}{3}\%$ farther than B drove. How far did B drive?

7. A merchant buys goods for \$480 and sells them for \$540. What is his per cent of gain?

8. A broker buys stock for \$4800 and sells it for \$5600. What per cent did he gain?

9. Mr. Jacobs buys goods for \$39 and sells them for \$52. What per cent did he gain?

10. Mr. Gould buys a lot for \$1200 and sells it for \$2400. What per cent did he gain?

11. A raised 80 bushels of potatoes, which was $16\frac{2}{3}\%$ less than B raised. How many bushels did B raise?

12. If I buy a horse for \$120 and sell it for \$140, what per cent do I make?

13. A received a salary of \$720 and spends $66\frac{2}{3}\%$ of it. How much did he spend?

14. If \$49 is $87\frac{1}{2}\%$ of my wages, what are my wages?

15. If $37\frac{1}{2}\%$ of my money is \$33, how much have I?

16. A merchant makes a profit of \$4200 from his business and spends $33\frac{1}{3}\%$ of it for family expenses. How much does he save?

17. A merchant sold 24 dozen eggs, which was $37\frac{1}{2}\%$ of all he had. How many had he?

18. A farmer had 24 hogs in one pen and 12 in another. He took 4 hogs from the first and put them in the second. What per cent decrease in the first pen? What per cent increase in the second?

19. He again took from the first 4 hogs and put them in the second. What per cent increase in the second and what decrease in the first?

20. Books that cost \$21 were sold so as to gain $28\frac{4}{7}\%$. Find the selling price?

21. Goods that cost \$88 were sold so as to gain \$77. What is the per cent of gain?

139. Finding 10 %, 1 %, 5 %, and $\frac{1}{2}$ %.

	Then,	
$\frac{1}{10}$ of $100\% = 10\%$	$10\% = \frac{1}{10}$	$30\% = \frac{3}{10}$
$\frac{1}{100}$ of $100\% = 1\%$	$1\% = \frac{1}{100}$	$70\% = \frac{7}{10}$
$\frac{1}{20}$ of $100\% = 5\%$	$5\% = \frac{1}{20}$	$90\% = \frac{9}{10}$, etc.
$\frac{1}{200}$ of $100\% = \frac{1}{2}\%$	$\frac{1}{2}\% = \frac{1}{200}$	

By Sec. 19, we learned that to find $\frac{1}{10}$, we should point off one decimal place or remove the decimal point one place to the left; to find $\frac{1}{100}$, we remove the decimal point two places to the left. Then to find 10 % or 1 % of any number, we remove its decimal point respectively one or two places to the left. Thus, 10 % of 33 is 3.3, and 10 % of \$54.20 is \$5.42. Also, 1 % of 256 is 2.56, and 1 % of \$5425 is \$54.25.

Since $\frac{1}{20}$ is $\frac{1}{2}$ of $\frac{1}{10}$, we may find $\frac{1}{20}$, or 5 %, by taking $\frac{1}{2}$ of $\frac{1}{10}$ of it. Thus, 5 % of \$976 is $\frac{1}{2}$ of \$97.60, or \$48.80. Likewise, $\frac{1}{2}$ % or $\frac{1}{200}$ is $\frac{1}{2}$ of 1 %, or $\frac{1}{2}$ of $\frac{1}{100}$. Thus, $\frac{1}{2}$ % of \$976.25 is $\frac{1}{2}$ of \$9.7625, or \$4.88.

PROBLEMS

1. A miller makes 56 bushels of wheat into flour in one day. If this is 10% of the wheat he has, how many days will it take him to grind all of it at the same rate?

2. James has $76, which is 5% less than I have. How much have I?

3. A man owed $6700 when he died. His property was worth 90% of this amount. What was the value of his property?

4. Mr. Berry sold a carriage for $195, which was 30% more than it cost him. What did it cost him?

5. A merchant sold goods that cost him $34 at a loss of ½%. Find the selling price.

6. A house worth $6000 is rented for 5% of its valuation. What is the rent?

7. A bedroom suite was sold for $77, which was 10% above cost. Find the cost.

8. A farmer sold two cows for $38 each, gaining 5% on one and losing 5% on the other. Find the cost of each. Find the loss or gain in the transaction.

9. An implement dealer buys a wheat drill for $54, and sells it at a profit of 10%. What is the selling price?

10. A young man had $600 and received $4200 by will. What per cent was the increase in his wealth?

11. A ton of coal was sold for $9, which was a gain of 12½%. Find the cost.

12. A father had $1400. He gave 25% to his son, 40% of the remainder to his elder daughter, and the remainder to the younger daughter. How much did each receive?

13. A merchant sells $56 worth of goods to-day, which is 12½% less than he sold the day before. Find amount of his sales the day before.

14. A grocer buys a hogshead (56 gal.) of molasses and sells 30% of it the first day. How many gallons had he left?

15. Mr. Burkett sold a buggy for $85, which was 70% more than it cost him. Find cost.

16. I loan $600 and receive $30 interest. What per cent is this of the amount loaned?

17. A commission merchant charges $20 for selling $4000 worth of wheat. What per cent is that of the selling price of the wheat?

140. Finding Other Per Cents. Much the larger part of all business problems involving percentage has to do with per cents given in Sections 137–139. Most of the problems in interest, bank discount, and trade discount may be solved with them or with slight modifications of them, and these are the most commonly used of all percentage applications. A thorough mastery of the per cents thus far developed, then, is of first importance, and they should be practiced upon until problems involving any phase of these per cents can be solved quickly and with accuracy.

Finding any per cent, other than those mentioned, is based on the method of finding 1 %. Thus, 7 % is $\frac{7}{100}$, or 7 times 1 %; 13 % is $\frac{13}{100}$, or 13 times 1 %; 34 % is $\frac{34}{100}$, or 34 times 1 %; etc. To find any given per cent first find 1 % by removing the decimal point two places to the left, and then multiply by the number of per cent required.

Thus, 4 % of \$3296 is \$32.96 × 4 = \$157.04.

PROBLEMS

1. A farmer raised 450 bu. of potatoes and sold 6 % of them. How many bushels had he left?

2. I spent \$28, which was 14 % of my money. How much had I at first?

3. I have \$65 and spend 12 % of it for trousers. What do they cost?

4. A buys goods for \$36 and sells them for \$39. What is his per cent of gain?

5. Mr. Bunger sells his horse for \$65 and by doing so gains 8½ %. What was the cost?

6. If Street & Co. pay \$45 for dishes and sell them for \$48, what per cent gain have they?

7. If a coal dealer bought coal for \$7.50 and sold it for 6⅔ % advance, what did he gain?

8. Sold goods so as to gain \$36, which was a gain of 12 %. What was the cost? What the selling price?

9. I sold goods for \$376 and lost 6 %. Find the cost.

10. I have 320 sheep and buy 8 more. What per cent do I add to my flock?

11. I have $39 and spend $21. What per cent do I spend?

12. A merchant paid $17 for goods and sold them so as to gain $3. What per cent did he make?

13. A man spent 28% of his money for a coat that cost $56. How much money did he have?

14. A miller lost 36% of his wheat by fire and had 1280 bushels remaining. How many bushels did he lose?

15. A had 240 chickens and sold 84 of them. What per cent of his flock had he left?

16. A dealer sold grain at a profit of 16% and received $696. What did it cost?

17. A speculator sold land for $9200, which was 8% less than it cost him. What did it cost him?

18. A stockman bought horses for $84 and sold them for $96. What per cent did he gain?

19. I sell goods for $184 and lose 8%. What did they cost?

20. A man sold his horse for $147, which was $33 less than he paid for it. What per cent did he lose?

GENERAL PROBLEMS

1. If a grocer adds a pound of Java coffee to every four pounds of Mocha, what per cent of the mixture is of each coffee?

2. A man owning $\frac{1}{4}$ of a mill, sells $\frac{1}{4}$ of his share. What per cent of the mill does he still own?

3. A man's money invested at 10% annual interest yields $125 a month. How much has he invested?

4. My agent sold $625 worth of goods and charged me $\frac{1}{2}$% commission for selling. How much money did I pay him?

5. A mechanic has $42 a month left after paying 6$\frac{1}{2}$% of his wages for car fare. What are his wages? How much does he pay for car fare?

6. Parker & Co. pay $60 for a buggy and sell it for $64. What per cent do they make?

7. A hardware man sold a stove for $84 and lost, by so doing, 6$\frac{1}{4}$%. What was the cost of the stove?

8. A jeweler sold a watch so as to gain \$2, which was $2\frac{1}{2}\%$ of the cost. What was the cost?

9. The annual interest on my money loaned at 4% is \$75. How much have I loaned?

10. A man invested 60% more money in a business than his partner, and the difference between their investments was \$3000. How much did each invest?

11. A man bought a horse and a cow for \$300, the horse costing 50% more than the cow. How much did each cost?

12. $\frac{4}{5}$ of James's money is 75% of Henry's, and $\frac{2}{3}$ of Henry's is 25% of John's. John has \$32. How much has James?

13. A capitalist had a half interest in a ranch and sold $\frac{1}{3}$ his interest for \$3600. The sale was made at a gain of 25%. What was the cost of the ranch?

14. A bank building rents for \$4200 a year, which is $12\frac{1}{2}\%$ of its value. What did it cost, if it had increased in value 40%?

15. If a firm quits business with property worth \$2600 and owes \$3900, what per cent of their debt can they pay? How many cents on the dollar?

16. If I sell \$6000 worth of goods for my principal and remit him \$5900, what per cent commission did I charge him?

17. A farmer lost $16\frac{2}{3}\%$ of his hogs, sold 40% of the remainder, and had left 120. How many had he at first?

18. A man bought property which increased in value $16\frac{2}{3}\%$ the first year. He finally sold it at an increase of 25% over this, and received \$8400. What was the cost?

19. A lady bought a piano for \$480, which was 25% of the money she had in the bank. The money she had in the bank was 40% of the value of real estate that she owned. What was the value of the real estate?

20. A confectioner sold candy for 60¢ a pound. If this were 25% more than it cost him, what was the cost?

21. A clothier sold a suit of clothes for \$18 and lost 40%. He then sold another at a profit of $16\frac{2}{3}\%$ and gained as much as he had lost on the first. What was the cost of each suit?

22. I sold a house and lot for \$1600, losing 20%. For how much should I have sold it to gain 20%?

23. A merchant engaged in business, investing cash $3500. At the end of one year he found that he had paid for merchandise, $3250; for rent, $450; for clerk hire, $1200; for incidentals, $725. He had sold merchandise to the value of $6758.40. What was his per cent of profit on his investment?

24. A man's real estate is now worth $12,000. The first year it increased 20% in value, and the second year 33⅓%. What did it cost two years ago?

25. A dry goods merchant marked cloth at 25% advance on the cost, but was obliged to sell it at 20% less than the marked price. If it cost him $1 a yard, what did he sell it for?

26. A man willed 50% of his property to his wife, 60% of the remainder to his invalid daughter, and the remainder to his church. The church received $650. What did the wife and daughter receive?

27. A hardware dealer engaged in business and lost 12% of his money the first year and gained 33⅓% the second year. If he started with $4500, how much has he now?

28. If a man loses $2400 by selling at a loss of 12½%, at what should he sell to gain 12½%?

29. Mr. McFarland bought a square piece of land, containing 40 acres, paying $800 for it. He opened a street through the center of it and divided the land on each side into lots 4 rods wide. If he sold the corner lots for $540 apiece, and the other lots for $400 apiece, and the expense of opening up the land was $1250, what was his per cent of profit or loss?

30. A speculator bought a section of land for $3 per acre. He sold ¼ of it at $8 per acre, and the remainder at $9.50 per acre. If the cost of making the sales was $135, what was his gain per cent?

31. A merchant's profit the second year was 66⅔% greater than it was the first. The profits of the two years amounted to $8120. What was the profit of each year?

32. I offered my house and lot for sale at 50% above cost, but sold them for 25% below the asking price and gained $600. What was the cost? What the gain per cent?

33. If my wheat cost 90¢ a bushel and I lose 10% by shrinkage, for what must I sell it to make a net gain of 12½%?

34. If I sell ½ of my land for what ¾ of it cost me, what per cent do I gain?

35. I paid $22,500 for two houses. If 75% of the cost of the one is equal to 150% of the cost of the other, what did each cost?

36. A has 25% more money than B, B has 20% more than C, C has 12½ less than D. How much has each, if together they have $33,900?

37. I paid $580 for a horse, wagon, and harness. The wagon cost 40% less, and the harness 66⅔% less, than the horse. What was the cost of each?

38. A miller in 3 years made gains amounting to $6336. The second year's gain was 20% greater than that of the first, and the third 10% greater than the second. What was each year's gains?

39. A grocer bought apples at 60¢ a bushel, and marked them so as to sell at a gain of 20%, but sold them at a reduction of 12½% from the marked price. If he gained $42.80, how many bushels had he?

40. A man sold goods that cost $425 at an advance of 40%. He lost 25% in bad debts and paid 5% for collecting. What was his gain or loss?

41. Our stock of goods decreased in value 33⅓%, and again 20%. It then increased 20%, and again 33⅓%, and was sold at a loss of $66. What was it worth at first?

42. If the retail profit is 33⅓%, what do I make on goods that cost $180, if I sell them at wholesale for 10% less than at retail?

43. Last year a merchant gained $2000. This year he gained 20% more, which is 44⅘% of what he gained the year before last. What did he gain each year?

44. A barrel of cider had lost 20% by leakage and was sold for 50% above cost. What per cent gain was that?

45. I buy a barrel of vinegar, containing 52 gallons, at 20¢ a gallon. If four gallons leak out, for what must I sell the remainder per gallon to gain 25%?

46. If I buy stocks for 80% of their value, and sell them for 110% of their value, what per cent do I gain?

47. Sold a lot of cotton at a gain of 33⅓%. With the money I bought another lot and sold it for $480 at a loss of 20%. What did the first lot cost me?

48. A manufacturer sold at a profit of 33⅓% to a wholesale dealer, who sold at a profit of 25% to the retail dealer. The retail dealer sold

at a profit of 20% and received $60 for the article. What was the original cost?

49. A merchant increased his investment 50%. He then withdrew 66⅔% of his capital and invested it in bonds. He now has $4800 in the business. How much did he invest at first?

50. An implement man bought a mower for $42. How much must he ask for it in order to make a discount of 25% and still gain 16⅔%?

51. I buy goods and sell at a loss of 10%. I reinvest the money and gain 10%. What per cent do I gain or lose?

52. A dairyman sold two cows for $120, and gained 20% on the one and lost 20% on the other. What was the cost of each, if he sold the first for 50% more than the other?

53. A real estate dealer sold a building for $15,000 and lost 40%. He sold a house and lot at the same time and gained 16⅔%. He did not gain or lose on the two transactions. What was the cost of each property?

54. A man bought a business for $9775, which was 15% more than the former owner paid for it. He then sold it at a profit of 6%. What was the selling price? What the gain? What the original cost?

55. The imports of sugar and molasses into the United States in 1891 amounted to $108,387,388; in 1900 they amounted to $101,100,000. What was the per cent of decrease?

56. The exports of wheat and flour in 1891 and 1900 were as follows: 1891, $106,125,188; 1900, $140,997,966. What was the per cent of increase?

57. The world's supply of sugar in 1900 was 8,800,000 lb. The supply of cane sugar was 2,850,000 lb. What per cent of the total supply was cane sugar?

58. The exports of the United States in a given year were: Europe, $697,614,106; Asia and Oceania, $43,813,519; British North American Possessions, $37,345,515; West Indies, $33,416,178; South America, $33,226,401; Mexico and Central America, $21,236,545; Africa, $4,738,847; all other, $879,172. Find what per cent of the total was exported to each country.

59. A wholesale dealer had sold $1500 worth of goods to a retailer. The retail dealer failed and could pay only 75¢ on the dollar. If the wholesale dealer paid 5% for the collection of the debt, what was his per cent of loss?

60. The estimated total cut of lumber in the United States from 1880 to 1906 was 706,712,000 board feet. Of this amount, Michigan produced 93,436,000. Michigan's output was what per cent of the total?

61. The freight moved over the several lines of railroad in the United States in a given year was as follows:

Class of Commodity.	Tonnage reported as originating on Line	Per Cent of Aggregate
Products of agriculture	56,102,838	?
Products of animals	15,145,297	?
Products of mines	269,372,556	?
Products of forests	60,844,933	?
Manufactures	71,681,178	?
Merchandise	21,697,693	?
Miscellaneous	26,493,338	?
Grand total	?	?

Find the aggregate tonnage. Find the per cent to two decimal places that each item is of the aggregate.

62. If the exports of manufactured products from the United States is as shown in the accompanying square, find what per cent of the aggregate is exported to each division.

Europe, $215,000,000
North America, $96,000,000
Asia, $34,000,000
Oceania, $29,000,000
South America, $27,000,000
Africa, $11,000,000

63. The total imports into the United States for a certain year were $903,320,948. Of this amount, the United Kingdom sent 18%; Germany, 11%; France, 9%; Brazil, 8.7%; British North America, 5.3%; all other countries, 48%. Find the amount sent by each division to approximate millions, and represent it in graphical form as in Problem 62.

64. From the accompanying graph, showing the per capita consumption of coffee in pounds, find the per cent of the whole amount consumed by each of the five countries, the population of the United Kingdom being approximately 43,650,000; of Italy, 33,750,000; of Austria-Hungary, 47,000,000; of Germany, 60,650,000; and of the United States, 90,000,000.

United Kingdom	.72
Italy	.98
Austria Hungary	2.04
Germany	4.62
United States	10.79

65. If the per capita consumption of sugar in Russia is 13.9; in Portugal, 14.2; in Austria, 17.6; in Belgium, 23.0; in Netherlands, 34.3; in France, 36.9; in Norway and Sweden, 40.6; in Denmark, 48.7; in Switzerland, 52.0; in the United States, 65.2; in the United Kingdom, 91.1 pounds, find total amount consumed by all, the population of each being, in approximate millions, as follows: Russia, 107½; Portugal, 5½; Austria, 26; Belgium, 6¾; Netherlands, 5½; France, 39; Norway and Sweden, 7½; Denmark, 2.6; Switzerland, 3½; United States, 90; and United Kingdom, 43.6. Find the per cent of this amount which each consumes. Represent in graphical form as in Problem 64.

XIV

TRADE DISCOUNT

141. Trade Discount is an allowance from the price of goods made because of special conditions, or for settlement of the account within a specified time. Such discounts are usually made "to the trade" or to persons engaged in the retail trade in a given line of goods, hence the term. The amount of the allowance is expressed in per cent or fractions. Discount plays a large part in the arithmetic of mercantile business.

142. Marked and List Prices. Merchants generally *mark* or *list* their goods, so they may gain a certain per cent of the cost as profit. If strictly a "one price" house, the per cent of profit is the only element to be considered by a retailer in determining the "marked" price. If the goods are to be so marked that a discount may be allowed and still a certain per cent of profit be made, then both discount and profit must be considered.

Wholesalers and manufacturers very generally publish a "list price" from which a certain per cent is allowed as a discount "to the trade." Fluctuations in the market prices may in this way be provided for by varying the discounts quoted instead of changing the printed or list prices. Goods listed and not subject to discounts of any kind are usually marked "net."

143. Term Discount. Even when goods are billed at such

discounted prices, a further discount is often allowed if the bill is paid within a certain time, say 30 or 60 days; while that discounted price may be still further discounted for *cash*.

Thus, goods may be listed at \$60, and quoted at discounts of 20, 10, and 5. The goods would usually be billed at "20 off" (\$60 less $\frac{1}{5}$), or \$48. 10 % off for payment within a time period marked, say 30 days, would reduce the bill (\$48 − \$4.80) to \$43.20, and 5 % further discount for cash would make the cash value of the bill $\frac{1}{2}$ of \$4.32 (\$2.16) less, or \$41.04.

When discounts are quoted in series as above, it should be noted that the discount indicated by the first per cent given, is taken from the amount of the bill, then the second discount is computed upon the remainder and deducted, and so for the third, each discount being a per cent of the remainder after deducting the previous discount.

144. Fractional Discounts Used. Whenever the nature of the business will allow, prices are listed so that the discounts used are those per cents which are the equivalents of simple fractions. Thus, instead of allowing a discount of 15 %, the list price is raised slightly so that a discount of either $16\frac{2}{3}$ % or 20 % may be given. Finding $\frac{1}{6}$ is not only purely a mental operation, but the process is simpler, shorter, and less liable to error than multiplying $\frac{1}{100}$ of the amount by 15. For these reasons, probably, the trend of business usage for years past has been away from a purely decimal to a fractional discount.

145. Finding Net Price. 1. 200 arithmetics were billed to a dealer for \$80, on which a discount of 20 % was allowed. What was the net cost?

$$20\,\% = \tfrac{1}{5}$$
$$\tfrac{1}{5} \text{ of } \$80 = \$16$$
$$\$80 - \$16 = \$64, \text{ net cost.}$$

2. A discount of $25 was allowed on a bill of goods for $150. What was the per cent or rate of discount?

$25 = $\frac{25}{150}$ of $150.

$\frac{25}{150}$ = $\frac{1}{6}$ or $16\frac{2}{3}$ %, rate of discount.

146. Net Price with a Discount Series. 1. Discounts of 25, 20, and 5 per cent were quoted on a bill of goods amounting to $360. What was the net price?

$360.	= the list price.
90.	= 25 % or $\frac{1}{4}$ of $360.
270.	= price after first discount.
54.	= 20 % or $\frac{1}{5}$ of $270.
216.	= price after second discount.
10.80	= 5 % or $\frac{1}{20}$ of $216.
$205.20	= net price.

2. A bill of goods netted $216, after being discounted at 25 % and 20 %. What was the face of the bill?

$216 = 20 % or $\frac{1}{5}$ less than the amount of the last discount.

$\frac{4}{5}$ = $216.

$\frac{5}{5}$ = $\frac{5}{4}$ of $216, or $270.

$270 = 25 % or $\frac{1}{4}$ less than face of bill.

$\frac{3}{4}$ = $270.

$\frac{4}{4}$ = $\frac{4}{3}$ of $270 or $360, face of bill.

147. Single Equivalent of Discount Series. 1. Discounts of 25 % and 20 % are equivalent to what single discount?

100 %	= the whole of anything.
25 %	= $\frac{1}{4}$ of 100 %, or first discount.
75 %	= price after first discount.
15 %	= 20 %, or $\frac{1}{5}$ of 75 %.
60 %	= net price after second discount.

100 % = 60 % = 40 %, equivalent single discount.

2. A bill of goods netted $216, after being discounted 25 and 20 per cent. What was the face of the bill?

Discounts of 25 and 20 = 40 % discount (Prob. 1).

100 % − 40 % = 60 %, net price.

60 %, or $\frac{3}{5}$ = $216.

$\frac{5}{5}$ = $\frac{5}{3}$ of $216, or $360.

PROBLEMS

1. A merchant bought a bill of goods amounting to $400. He received a discount of 10% and 8⅓% off for cash. What was the cash value of the bill?

2. Fouts & Co. bought a bill of goods amounting to $600, on which they received a discount of 20% and 10% off for cash. What was the cash value of the bill?

3. A carriage is marked at $100, but the dealer tells me I may have it on 30 days' time at a discount of 20%, and, if I pay cash, an additional discount of 2½% will be given. What must I pay for it in cash?

4. The marked price of a mower is $64. I sell it to a farmer, giving him a discount of 12½% and 14⅞%. What did he pay?

5. I buy a bill of goods amounting to $500, on which I get a discount of 12½% and 4% off for cash. What did I pay?

6. Smith & Co. buy a bill of goods amounting to $500, on which they get a discount of 16⅔% and 10% off for cash. What did they pay?

7. McKee & Co. sell goods for $120. They had given a discount of 20% and 5% off for cash. What was the list price of the bill?

8. What is the cash value of a bill of goods that lists $49, if a discount of 14⅞% and 16⅔% is given?

9. Mr. Kerr sells a wagon on 60 days' time for $72. If paid in 30 days, the buyer will get a discount of 16⅔%; if he pays in 10 days, he will get 10% additional. What would the "cash in 10 days" payment be?

10. A dealer bought goods at 20% and 25% off, and sold them at 10% and 10% off the same marked price. What was his per cent of gain?

11. How much better is a series of 25%, 20%, and 10% than a single discount of 60%?

12. What is the amount of the following bill if it is paid within 10 days?

Chicago, Ill., May 2, 19—

Messrs. Chase & Witherspoons

Andover, Mich.

Bought of **W. M. WELCH & COMPANY**

119 Illinois Street

Terms: 6 da. net; 3%/10 da.

	20 hf. chts. Japan Tea	1200#	25¢		
	20 " " Oolong Tea	1000#	45¢		
	16 cases Ceylon Tea	800#	35¢		
	10 bales Mocha Coffee	1500#	24¢		
	20 mats Java Coffee	1500#	25¢		

13. Sold Lew A. Wallace, terms 5/10–n/30; 12 bx. Apricots @ $2.15; 12 bx. Layer Raisins @ $2.95; 5 bx. Dried Citron @ $1.33⅓; 3 tierces Refined Lard, 1065 #, @ 9¢; 3 cases Shredded Codfish, 6 doz., @ $1.12½; 4 cases Salmon, 8 doz., @ $2.12½. Find cash value.

14. Sold A. O. Bennett, terms 3/10–n/60; 4 cases Rio Coffee, 400 #, @ 17½¢; 2 bags Mocha Coffee, 185 #, @ 28¢; 2 bags M. G. Coffee, 378 #, @ 22¢; 4 bbl. Herring @ $6.25. What is the cash value of the bill?

15. Sold W. B. Lambert, terms 10–2/10–n/30; 5 doz. Baseballs @ $15; 3 doz. Bats @ $9; 24 Infielder's Gloves @ $2.50; 3 Catcher's Gloves @ $6.25; 12 Baseball Uniforms @ $14; 12 pr. Shoes @ $3.50. What is the cash value of the bill?

16. Bought of J. A. Amsbaugh & Co., 225 bbl. Flour @ $4.45; 165 S. H. Hams, 1960 #, @ 14½¢; 5 tierces Refined Lard, 745 #, @ 9¢; 6 bbl. Mess Pork @ $15.20; 8 bbl. G. Flour @ $6.10; 24 bx. Figs @ $1.85; 5 cases G. Coffee, 300 #, @ $0.12½. I am allowed a discount of 20% and an additional discount for cash of 5%. What is the cash value of the bill?

17. A man desires to buy 48,000 ft. of pine lumber. One firm offers the lumber at $60 per M, less discounts of 20% and 5%; another firm

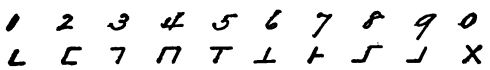
offers the lumber at $75 per M, less discounts of 33⅓% and 6⅔%. The terms offered by both are 1/10–n/30. Which is the better offer, and how much does the lumber cost?

148. Marking Goods for the Shelf. In determining the selling price of goods, it is usual to add to the cost a certain per cent as profit. Thus, if a set of books, costing $3.60, is to be sold at a profit of 25%, they must be marked at ¼ more than they cost, or $4.50.

In marking goods for sale, merchants often place the cost price as well as the selling price on each article. For this cost price a "cipher" is often used. Any word or phrase of ten letters, or any ten arbitrary characters, may be used as a cipher. Thus,

1	2	3	4	5	6	7	8	9	0	
i	m	p	o	r	t	a	n	c	e	Repeaters x and y.

or



Repeaters # +

When a figure occurs twice or more in succession, a repeater is used instead of repeating the character for the figure. This renders deciphering more difficult.

The cost and selling price may both be written on the tag. If so, the selling price is written below a horizontal line, and may be in cipher or plainly written.

 or $\frac{1.40}{1.75}$

Sometimes the cost price is written in an entirely different cipher, known only to the buyer or proprietor.

$\frac{\text{mon}}{\text{pmr}}$ or $\frac{2.48}{3.25}$

149. Listing Goods for Catalogues. Goods to be listed in a catalogue are usually so priced as to permit a discount or a discount series, and still to sell at a price which will afford the desired per cent of profit.

Thus, if the set of books (Sec. 148) costing \$3.60 is to be so listed that a discount from the list of $16\frac{2}{3}\%$ may be given and still yield a profit of 25%, the selling price of \$4.50 (Sec. 186) is $\frac{1}{6}$ less than the list price, or $\frac{5}{6}$ of list. The list price, then, would be $\$4.50 \times \frac{6}{5}$, or \$5.40.

If it were desired so to list an article, costing \$3.92, that a discount of 25%, $16\frac{2}{3}\%$, and 2% might be given, and still provide for a profit of 25%, the steps in the process would be as follows:

$\frac{5}{4}$ of \$3.92 = \$4.90, selling price.

\$4.90 = 98% or $\frac{49}{50}$ of price discounted 2%.

$\$4.90 \times \frac{50}{49} = \$5.00$.

\$5.00 = $\frac{5}{6}$ of price discounted $16\frac{2}{3}\%$.

$\$5.00 \times \frac{6}{5} = \$6.00$.

\$6.00 = 75% or $\frac{3}{4}$ of list price.

$\$6.00 \times \frac{4}{3} = \$8.00$, list price.

PROBLEMS

1. I buy goods for \$24 and desire to sell them so I may make $33\frac{1}{3}\%$. What must I ask for them that I may give a discount of 20% and still make my desired gain?

2. What must goods that cost 20¢ a yard be marked so as to make 25% after a discount of $16\frac{2}{3}\%$?

3. What must I ask for goods for which I paid \$16, if I desire to give a discount of 20% and still gain 25%?

4. A book seller bought a set of encyclopedias for \$40. He marked them so as to gain 10% and give a discount of 20%. What did he ask? What did he get?

5. A hardware merchant buys stoves for \$24 each. He desires to sell them so as to make $12\frac{1}{2}\%$ after deducting 25%. What did he ask? What did he get?

6. I wish to sell goods at cost that cost me \$400. In order to do so, I mark them up 25%. What per cent discount can I give?

7. I buy plows at \$9.60. If I wish to make a gain of 25% and still offer a discount of 25% and 10%, what must I ask for them?

8. A dealer marked a carriage by mistake at $160, and sold it at a discount of 25%, thereby losing 16⅔%. What should he have marked it in order to have gained 25%?

9. Wishing to sell goods at cost that cost me $5, I mark them at 20% advance. What discount must I give?

10. A merchant buys goods for $1200. What must he ask in order to give a discount of 16⅔% and still gain 10%?

11. How must goods that cost 25¢ be marked so as to make 20% and still give a discount of 16⅔%?

12. Find the list price, if goods that cost $600 are sold at a gain of 25% after a discount of 20%, 5%, and 2%.

Using the characters in Sec. 148, mark the cost and selling price of the following articles:

13. Shoes costing $5 and selling for $6.50.

14. Gloves costing $22.50 per dozen and selling at 20% gain.

15. Caps costing $7.25 per dozen and selling at 33⅓% gain.

16. Shoes costing $1.97 and selling at 25% gain.

17. Boots costing $2.68 and selling at $3.75.

18. Make a key from the letters contained in the words "blacksmith" and "authorizes," and mark the articles given in the above examples.

XV

COMMISSION

150. The Commission Business. In the development of our industries, certain cities become the centers for handling particular products. Thus, Chicago is a great grain market; and Chicago, Kansas City, and Omaha are great live-stock markets.

Where important products are handled so largely, there will naturally be a large number of buyers, and sellers will ship their products from the tributary territory. It would be too expensive to accompany their shipments to market, and yet they want their goods sold to the highest bidder. They must, then, have some one to represent them. This representative or agent of the owner of the product is known as a *commission merchant.*

These merchants receive the goods, see that they are properly cared for, effect a sale, and return the proceeds to the owner, less expenses in handling and a fee for the services rendered. This fee is usually computed as a per cent of the amount of the sale, and is called a *commission.*

151. Terms Used. Goods sent to an agent for sale are called *shipments*, and are received by the agent or *consignee* as a *consignment*, the title to the goods remaining with the *consignor.* The statement of the goods received, their sales in detail, with the expenses and charges thereon, showing the net proceeds, is called an *account sales.*

In a like manner commission merchants often receive

orders to buy, as agents, certain goods. For this service they are paid a commission. A detailed statement of the purchases, with expenses, charges, etc., is called an *account purchase.*

152. Commission in General. In general any percentage of money or value handled, which is received by an agent for his services as such agent, is a commission. Thus, a *real estate* agent receives a commission on property sold; an insurance agent a commission on insurance written; a collector, a commission on money collected, etc.

153. Commission on Purchases. Sometimes, an amount of money is sent an agent with instructions to expend it in the purchase of goods. The present tendency in business is to charge commission on the whole amount sent (gross). But many commission houses still charge only on the purchase (net). When the amount sent is to include the sum used for the purchase, together with the commission, it is equal to 100 % of the purchase to be made, plus the rate to be paid as commission on such purchase.

Thus, if $367.50 were sent to a commission merchant with which to buy wheat, after deducting his commission of 5 %, what would he invest in wheat?

The amount of money he received would include 100 % of the value of the wheat purchased and 5 % of such purchase as commission. $367.50 would then equal 105 % of the amount to be expended on wheat. $105\% = \frac{21}{20}$. Then $\$367.50 = \frac{21}{20}$ of the wheat purchase money. $\frac{1}{20}$ would be $\frac{1}{21}$ of $367.50, or $17.50, the commission, and $\frac{20}{20}$ would be $350, or the money for the purchase of wheat.

PROBLEMS

1. How many pounds of coffee, at 27¢ per pound, can be bought for $8424, if the agent is allowed 4% of purchase price for buying?

2. I remitted $2612.90 to a New York agent for the purchase of gloves. If the agent's commission is 4% net, and he makes an added charge of 1% for guaranteeing quality of goods, how many dozen pairs of gloves, at $8.50 per dozen, should he send me?

3. I remitted $600 to an agent for the purchase of peaches. If the agent's charges were 5% net for purchase and $12 for inspection, how many baskets did he buy at 43¢?

4. Rule a sheet of paper, copy the following account sales, making all necessary extensions, etc.

Chicago, Ill., April 20, 19—

Sold for the account of

F. W. Martin, Freeport, Ill.

By HOWARD PAYNE & COMPANY
COMMISSION MERCHANTS
150 Randolph Street

19—						
Mar.	12	200 bbl. Gold Medal Flour	6.50			
	21	150 " Pillsbury's Best Flour	6.75			
Apr.	10	240 " C. W. Flour	5.60			
	15	125 " R. P. Flour	4.60			
		Charges				
Mar.	10	Freight 145.00	Cartage 32.00			
		Storage 17.50	Insurance 6.75			
		Guaranty 2%	Commission 4%			
		Net Proceeds				

5. Prepare an account sales under date of May 15, 1910 for 6000 bu. Wheat, 3000# Beef, sold Westerfield Bros., Greenville, O., for the account of W. C. Pierse & Co., Union City, Ind., Sales: April 10, 3000 bu. Wheat @ 62½¢, 1500# Beef @ 9½¢; May 12, 3000 bu. Wheat @ 65¢, 1500# Beef @ 10¢. Charges: freight, $125; cartage, $15; storage, $17.50; insurance, ¼%; commission, 2%.

6. Rule a sheet of paper and copy the following purchase, making all extensions, etc.

New York, N. Y., May 20, 19—

Purchased by

L. M. BARKER & CO.

For the account and risk of

A. M. Randall & Co., Richmond, Ind.

6	1/2 chs. G. Tea	330#	34¢					
8	" O. Tea	450#	45¢					
10	" J. Tea	700#	25¢					
16	mats R. Coffee	1200#	23¢					
	Charges							
	Cartage		7.50					
	Commission 2½%							
	Amount charged to your account							

7. Prepare an account purchase for coffee purchased by E. H. Reed & Co., June 15, for the account and risk of Ames, Spencer & Co. Purchases: 8 mats G. P. Coffee, 600 ℔ at 23¢; 16 mats M. Coffee, 1300 ℔ at 34¢; 24 mats J. Coffee, 1800 ℔ at 30¢. Charges: cartage, $6.70; commission, 2% net.

8. A man places a claim of $2500 in the hands of an attorney for collection. If the debtor is a bankrupt, having liabilities to the amount of $20,000 and resources to the amount of $15,000, how much should the creditor receive after a commission of 2½% is deducted for collection?

9. A principal sends his agent $2500 with which to buy corn. After deducting his commission of 2½% on the amount sent, he buys corn, paying 45¢ a bushel. How many bushels does he buy?

10. Find the rate of commission and guaranty on a purchase if the total cost is $4,143.10, the commission $174.60, the cartage $40, and guaranty $48.50.

11. A land agent receives $54,000 with which to buy land. After deducting his commission of 5% on the amount sent, how many acres can he buy at $35 per acre?

12. I sent my agent $4080 with which to buy corn. After deducting his commission of 2% net, how many bushels can he buy at 54¢ a bushel?

13. A dairyman received from his city agent $980 as the net proceeds of a shipment of butter. If the agent's commission is 2½%, delivery charges $12.55, how many pounds at 25¢ a pound must he have sold? What was the agent's commission?

14. An agent remits to his principal $3656.25 after deducting a commission of $93.75. What was the selling price of the goods? What was the rate of commission?

15. An agent sells cotton for $3600 and charges 2% commission. After deducting his commission of 3% for buying, he buys corn at 30¢ a bushel. How many bushels does he buy?

16. Find the proceeds of a sale, if the agent charges 2½% commission, $17.50 for cartage, $10.45 for storage, and $4.25 for insurance. The net proceeds are $1545.74.

17. I sell goods for $3600 and charge 3% commission. I then invest the proceeds, less a commission of 3% on the proceeds, in grain. How much do I invest in grain?

18. An agent charged me 5% for selling wheat, and 2% for investing the proceeds in cotton. His commission was $558.90; what was the selling price of the wheat?

19. I sent my agent in Missouri $2000 with which to buy apples at $1.75 per barrel and to pay commission and drayage. His charges were: commission 4%, drayage 5¢ per barrel. How many barrels did he buy, and what was his unexpended balance?

20. Render in full the following account sales, supplying rates for insurance and commission: April 16, Jacob Speer & Co., Portland, Ore., sold Henry Williams & Co., Chicago, 12,000 Salmon at 11¢; 18,000 Halibut at 16¢. Charges: freight, $250; insurance, $32; commission, $128.

21. Render an account sales for the following sales, supplying all rates: The Henry St. Clair Co., Cleveland, O., sold for A. W. Raymond, Sandusky, O., 250 bu. Beans at $1.06; 400 lb. Cheese at 10¢; 40 bbl. Flour at $5.60; 1500 bu. Wheat at 65¢. Charges: freight, $84.60; storage, $24.30; inspection, $4; insurance, $3.76; commission, $75.20.

XVI

TAXES AND DUTIES

TAXES

154. Property Tax. Money levied by the government for defraying its expenses is called a *tax*. For local, county, and state purposes this tax is usually levied upon property.

An *assessor* lists all the property in his district and places a valuation upon it. This valuation is either an estimated actual cash value or a certain per cent of it, as may be determined by law or custom in the locality. There is usually a board of review or equalization in the local district, county, and state, which compares valuations of different classes of property made by the different assessors, and equalizes them. The total of all the property valuations constitute the *assessed valuation* of the township, village, school district, county, or state. An estimate is then made of the amount needed for the governmental expenses of each of such units, and the per cent such an amount is of the assessed valuation is the *tax rate* levied. This per cent is so often fractional that it is usually expressed in mills; meaning the number of mills paid as tax on each dollar of property valuation.

155. Kinds of Property. Property is of two kinds—*real estate*, or lands with their improvements and buildings, and *personal property*, which includes all other property. Assessments are made separately for each kind of property, and often the personal and real taxes are due and become delinquent at different dates. Where taxes become delinquent, a penalty is usually charged for their nonpayment on time.

156. Other Forms of Taxation. Taxes levied solely upon tangible property often lead to serious inequalities. Efforts to distribute the burdens of government according to the benefits derived therefrom and according to the ability to pay, have led to many forms of special taxation, such as taxes upon capital stock, gross earnings, or mileage of wires or railroad bed, etc. *Franchises*, granted by local legislative bodies, giving to private corporations the right to use the streets for pipes, rails, or wires, etc., are often taxed. *Inheritances* and *incomes* are also taxed in some countries and states. Revenue is derived, too, from the sale of *licenses*, from *fees* for official acts, and from *fines* imposed for breaches of law. In some states a tax is laid upon each voter, regardless of property owned. This is a *per capita* tax, and it is known as a *poll* tax.

157. Finding the Rate of Taxation. *Example.* A township board finds that the assessed valuation of all property in the township is $600,000, and the total amount of revenue needed is $15,000. How much tax must be levied on each dollar of property valuation?

If $15,000 must be raised on a valuation of $600,000, the part taken for taxes will be $\frac{15000}{600000}$, or $\frac{1}{40}$, which is $2\frac{1}{2}\%$, or 25 mills, on the dollar.

158. Finding the Tax. *Example.* A man's assessment is $16,000. The total rate of taxation for all purposes in the city of his residence was 25 mills. What was his individual tax?

If he was to pay 25 mills on each dollar, he would pay $2\frac{1}{2}\%$, or $\frac{1}{40}$, of his property value as a tax.

$\frac{1}{40}$ of $16,000 is $400, amount of tax.

PROBLEMS

1. If property is valued at $500 and the taxes are $5, what is the rate of taxation?

2. If property is valued at $25,000 and a tax of $500 is to be paid on it, what is the rate?

3. The property in a town is valued at $986,600 and the amount of revenue to be raised $4933. What would be my tax if my property is valued at $5500?

4. The assessed valuation of a man's property is $12,500, and the rate of taxation is 23 mills on the dollar. What is his tax?

5. The real estate of a city is valued at $9,864,000. A revenue of $108,640 must be raised. The tax on personal property is $12,000. What will be the rate on real estate?

6. What will be a man's tax, at the above rate, who has property valued at $6400?

7. The value of the property in a township is $236,600 and the amount of money needed for state purposes is $709.80. What is the rate of taxation for the state? The rate fixed by the school board is 9.4 mills. What amount is collected for school purposes?

8. I have property valued at $9800 and the total rate of taxation is 64 mills. How much tax do I pay?

9. The personal property in a certain township is valued at $120,000 and the real estate at $2,400,000. The personal property is taxed at $\frac{1}{2}$ its value and the real estate at $\frac{1}{4}$ its value. If the rate is 25.8 mills, how much tax is collected?

10. The expenses of a certain town are $595,512.80. The tax rate is 16 mills on the dollar. What is the assessor's valuation?

11. The assessed valuation of a county as fixed by the board of equalization is $25,395,180. The rates of taxation are as follows: State, 3.5 mills; State University, 2 mills; State Agricultural College, 1.5 mills; State Normal School, 1 mill; County, 3 mills; County School, 1 mill; County Bridge, 3 mills; Insane Hospital, .5 mill; County Bond, 2 mills; County Poor, .2 mill; Soldier's Relief, .3 mill; County Road, 2 mills; Board of Health, .1 mill; Teacher's Fund, 9.4 mills (average); School Contingent Fund, 3.7 mills (average). What is the total tax raised in the county? What is the total rate of taxation? What is property taxed that is valued at $5400? What is the amount of tax levied for each purpose?

12. A tax of $6649.60 is to be raised in a certain city. The valuation of the real property is $898,500, which is assessed at $\frac{1}{2}$ its valuation. The personal property is assessed at $65,000. There are 1200 polls,

assessed at $2 each. What is the rate of taxation, the rate for collection being ¼%? What is a man's tax whose real estate is valued at $14,500, and personal property at $4500?

DUTIES

159. National Taxation. National revenue is raised largely by the forms of taxation known as *duties* and *excises*. Import duties are taxes levied upon goods imported into a nation. In the United States, Congress establishes a *tariff* or list of such duties. These duties are collected at the custom houses to be found at the various *ports of entry*. The officer in charge of the custom house is called a *collector*.

Import duties are of two kinds: those consisting of fixed charges on certain units of measurements of commodities, as the gallon, pound, or yard, called *specific* duties, and those which form a certain per cent of the value of the goods being imported, called *ad valorem*. Specific duties only are levied on certain articles, ad valorem only on others, and on some, duties of both kinds are levied. A statement or invoice, showing the various articles being imported, the distinguishing mark, number, or quantity and value of each, is termed a customs declaration or *manifest*.

When the goods are purchased in a foreign country for import, an invoice giving descriptions and prices is filed with the United States Consul in the district where the purchase is made, and he sends a copy to the collector of the port to which the goods are shipped. When the goods arrive, the cases are opened and the quantity and value as compared to that stated by the invoice is ascertained. For too great an undervaluation a fine may be imposed or the goods confiscated. Travelers' baggage is also inspected for dutiable goods.

Instead of paying duty at once, goods may be stored in a bonded warehouse, from which they may be withdrawn for

export or upon payment of the duty. Such goods are said to be *in Bond*. Importing goods without the knowledge of the government officers, thus avoiding the payment of duty, is called *smuggling*, for which heavy penalties are prescribed.

Excises are taxes levied upon certain specified articles, as liquors, tobacco, opium, oleomargarine, etc., which are grown or manufactured in this country. For the collection of these taxes, there are National Revenue districts, and collectors for each district.

TABLE OF CUSTOM DUTIES ON CERTAIN ARTICLES

ARTICLES	TARIFF RATE
Alcohol	¼¢ per lb.
Brushes	40% ad valorem
Carpets, treble ingrain	22¢ per sq. yd. & 40% ad valorem
Carpets, two-ply	18¢ per sq. yd. & 40% ad valorem
Carpets, tapestry, Brussels	28¢ per sq. yd. & 40% ad valorem
Carpets, Wilton, Axminster	60¢ per sq. yd. & 40% ad valorem
Cigars and cigarettes	$4.50 per lb. & 20% ad valorem
Cotton hosiery (made on machines and not otherwise provided for)	30% ad valorem
Cotton plushes, unbleached	9¢ per sq. yd. & 25% ad valorem
Cutlery, more than $3 per doz.	15¢ per piece & 35% ad valorem
Cutlery, razors, $3 or more per doz.	$1.75 per doz. & 20% ad valorem
Cutlery, sword blades	50% ad valorem
Dyewoods, common extracts of	⅛¢ per lb.
Glass, polished plate not over 384 sq. in.	10¢ per sq. ft.
Gloves, men's kid	$4 per doz. pair
Glue, value not over 10¢ per lb.	2½¢ per lb.
Hay	$4 per ton
Linen, wearing apparel	50% ad valorem
Silk, woven	45¢ per lb. and 60% ad valorem
Silk laces, wearing apparel	60% ad valorem
Skins, uncured, raw	Free
Velvets, cotton, unbleached silk	9¢ per lb. & 25% ad valorem
Wines, still, in casks containing more than 14% alcohol	60¢ per gal.
Woolen or worsted clothing	44¢ per lb. & 60% ad valorem

PROBLEMS

1. What is the duty on 60 yd. of silk, weighing 4 oz. to the yard, invoiced at $1.50?

2. What is the duty on a consignment of 26 doz. men's kid gloves, invoiced at $22.50 per dozen?

3. If silk laces are invoiced at 7540 francs, what will be the duty?

4. What will be the duty on an importation of $646 worth of skins?

5. What is the duty on an invoice of 642 lb. glue; 240 lb. alcohol; 20 doz. brushes, at 25¢ each; and 450 lb. extract of dyewood?

6. What is the duty on 14 doz. sword blades, worth $7.40 per dozen, and 16 doz. razors worth $4.50 per dozen?

7. For what must I sell hose, if they are invoiced at $2 per dozen, in order to gain 20% after paying the duty?

8. A merchant imported 1420 yd. of plush ($\frac{4}{5}$ yd. wide), invoiced at 2 francs a square yard. At what price per yard must he sell it to gain 25%, after paying $65 freight charges and the import duty?

9. What is the duty on 650 yd. Axminster carpet, 30 in. wide, if the invoice price is 40¢ per yard? For what must it be sold to gain $33\frac{1}{3}$%?

10. 1000 boxes (100 in each) of cigars weighing 1200 lb. net cost $13,675, delivered. If the freight and charges, other than the duty, amount to $45, what was the invoice price of the cigars?

11. After allowing 10% for leakage, a wine merchant paid $1768 duty on 24 casks of wine at $2 per gallon. How many gallons did the casks originally contain?

12. What will be the duty on 250 boxes plate glass, each containing 25 plates, 16 × 20 in.?

13. Find the amount of the duty and the total cost of the following:

MANCHESTER, ENGLAND, Jan. 14, 19—.

MESSRS. A. T. BOYD & CO.,
New York, N.Y.

BOUGHT OF WILLIAM & FIRTH

TERMS: 30 da.

W F		565 yd. Silk Velvet 1 12*s.* 3050 yd. Linen 4*s.* 3*d.* 1450 yd. Cotton Webbing 1*s.* 4250 yd. Wilton Carpet 12*s.* 6*d.*				

14.

MANIFEST No. 675 INVOICED AT BRADFORD, ENG., JUNE 12, 19—.

INWARD FORWARD ENTRY OF MERCHANDISE

Imported by GATES & WARMAN. In the Steamer Columbia.

AMOS PETERMAN, Master. From Liverpool, Eng.

Arrived July 15, 19—.

MARKS	No.	PACKAGES AND CONTENTS	QUANTITY	FREE LIST	VALUE	AD VALOREM DUTY	SPECIFIC DUTY	TOTAL DUTY
A.A.	1356	4 cases Woolen Clothing	2400 lb.		£342.12.0	?	?	?
	756	6 cases Cotton Hose	192 doz.		£74.10.0	?	?	?
					$	$	$	$

XVII

INTEREST

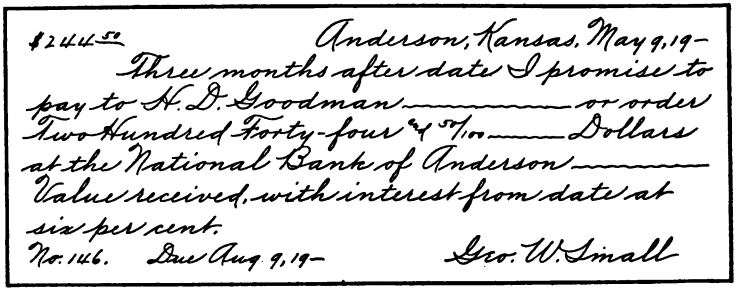
160. Loaning Money. When money is loaned, a charge is made for its use. This charge or rental for the use of money is called *interest.* Interest is always a certain per cent of the amount loaned, and the rate is the rental for a period of one year.

Thus, $5000 loaned at 6% would mean that 6% of the $5000, or $300, must be paid for its use each year. Of course, one half of that amount, or $150, would be the interest for six months, and twice that amount, or $600, would be the interest for two years, etc.

161. Notes and Mortgages. When loans are made, a written instrument or contract called a *promissory note* is signed by the person receiving the money. It is a written promise to pay the amount named at a certain time, usually with interest from date of the note, or from the date agreed upon for payment, called *maturity*, and contains an acknowledgment of the receipt of value therefor.

The payment of a note is often made more secure by the signature of other persons as guarantors and known as *indorsers*, or by conveyance of property, real or personal, conditioned upon the nonpayment of the note. When property is thus pledged, the instrument of conveyance is called a *mortgage.* If for personal property, it is a *chattel mortgage;* if for real estate, a *real estate mortgage deed.* In either case, the title only to the property is transferred, possession to be given upon nonpayment of the note, and upon the success

of legal proceedings known as foreclosure. The one who signs the note is called the *maker*, and the one to whom the note is to be paid is called the *payee*.



$244.50 Anderson, Kansas, May 9, 19–

Three months after date I promise to pay to H. D. Goodman ——— or order Two Hundred Forty-four 50/100 ——— Dollars at the National Bank of Anderson ——— Value received, with interest from date at six per cent.

No. 146. Due Aug 9, 19– Geo. W. Small

A PROMISSORY NOTE.

162. Bonds. For loans made to corporations, the note is called a *bond*. These may be either governmental, as city or county, or private, often called industrial bonds. In notes or bonds extending through a period of years, the interest is often provided for in separate notes for each interest payment, which are attached to the principal note, and are known as interest *coupons*. These bonds are usually issued for the purpose of raising money for improvements, development of property, etc. They pledge the property of the corporation as security, and are generally sold in the open market.

163. Compound Interest. In *Compound Interest*, unpaid interest is added to the principal at the end of each interest period. The whole amount is then considered as a *new principal*, upon all of which interest thereafter is to be paid. The interest and principal are thus compounded or united. Thus, $100 at 6% would amount to $106 at the end of the first year, to $112.36 at the end of the second year, to $119.10 at the end of the third year, etc. Money loaned at compound interest increases so rapidly as to more than double itself at 6% in 12 years.

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164. Periodic Interest. When interest due at the close of each interest period begins to draw simple interest until the obligation is paid, it is said to be *periodic* interest. That is, the interest on the principal for each time period draws interest; but this interest on the interest does not become due until the note is paid, and therefore does not itself draw interest; *i.e.* it is not compounded. Thus, a $100 note, if paid at the end of the first year, would amount to $106, at end of second year to $112.36, third year to $119.08, etc.

If the interest period is one year, it is *annual;* if six months, *semiannual;* and if three months, *quarterly interest.*

$100 at 6% simple interest would amount in ten years to $160; at annual periodic interest to $176.20; and at compound interest to $179.08.

165. Legal Limitations to Interest. When the rate of interest is not stipulated in the note or contract, the rate fixed by law is collectable. Such rate is known as the *legal rate,* and varies in different states and territories.

Most states establish a limit to the rate which may be charged even when agreed to by both parties. This is known as the *contract rate.* Interest charged in excess of the amount allowed by law is *usury.* While several states have no provisions regarding usury, most states forbid it. Various penalties are prescribed, such as the forfeiture of principal and interest, of principal, of interest, or of excess of interest over the legal rate.

The collection of compound interest cannot usually be enforced by legal process, but its payment or receipt is not illegal.

In some states periodic interest may be collected when in the contract, while in others it cannot be enforced. When each interest payment is put in the form of a note, as a coupon, it may be collected in any state and is not considered usury.

In order to prevent undue hardship, most states have, in the past, allowed a creditor three days after the stipulated date of maturity, before a note might be collected through legal process. These are known as *days of grace.* The custom, however, is dying out, and most states have repealed the law. Improved communication leaves less necessity for days of grace than formerly.

166. Common and Exact Interest. In computing the time of notes or other interest-bearing obligations, the method of compound subtraction (97) is used. It is the common practice to count a month as uniformly 30 days and the year 360 days. If the note reads "days after date," the date of maturity is usually computed to the exact day; if "months after date," then the same day of the month in the appropriate month.

The method still in use by the United States government and by the more conservative business men takes account of the exact number of days in the intervening calendar months, and the days are 365ths of a year. This is known as *exact interest.* Exact time between given dates and exact interest at any rate on various amounts for any number of dates, are usually found by reference to "Exact Interest Tables," with computations already made.

Interest found by the 60-day method, or by any method using 30 days as a month, counts a year as 360 days. It is, therefore, $\frac{5}{365}$, or $\frac{1}{73}$, greater than "exact" interest for the same period. Adding $\frac{1}{72}$ of itself to "exact" interest would likewise equal "30-day" interest.

167. Finding Interest. When the period of a note or other obligation is an exact number of years, the problem is one of simple percentage. If the interest rate is 6 % and the time three years, 18 % of the amount loaned would be the interest; if 8 % for 4 years, 32 % of the amount loaned, etc.

But most interest computations in business have to do with short-time loans and discounts. Hence they involve chiefly months and days. On account of the frequent occurrence of 6% as a rate of interest in business, and because of the simple relation 6 bears to 12 (the number of months in the year) and the easy fractional parts that relation makes in computing time, interest on any sum for a short period is most easily and accurately calculated as though at 6%, and then changed to the required rate.

168. Six Per Cent Interest for 60 Days. If every 12 months 6% of the amount of the loan must be paid as interest, every two months $\frac{2}{12}$ ($\frac{1}{6}$) of that, or 1%, must be paid. Then 1% of the amount loaned would be the interest for 2 months, or 60 days.

Thus, \$840 at 6% for 60 days is \$8.40; and \$1726 at 6% for 60 days is \$17.26. From this it will be seen that *the interest on any sum at 6% for 60 days is as many cents as dollars loaned;* the same is true of 4% interest for three months and 3% interest for four months. These rates are usable in savings banks and for interest on government bonds.

169. Six Per Cent for Any Time.

PROBLEM. — Find the interest on \$840 for 1 yr. 5 mo. 6 da.

\$8.40, interest for 2 mo.
\$67.20 (\$8.40 × 8), interest for 16 mo.
4.20 ($\frac{1}{2}$ of \$8.40), interest for 1 mo.
.84 ($\frac{1}{10}$ of \$8.40), interest for 6 da.
\$72.24, interest for 1 yr. 5 mo. 6 da.

To find the interest for years and months, first reduce to months. Then take such a simple multiple of the interest for two months as will make the required time, if for even months. If for odd months, add $\frac{1}{2}$ of the two months' interest

to the interest for one less than the required number of months.

For days, fractional parts of the interest for 2 months should be taken, using simple fractional parts wherever practicable and doing as much mentally as possible. Thus, the interest for 15 days is $\frac{1}{4}$ that for 60 days; for three days $\frac{1}{2}$ of $\frac{1}{10}$ of it; for 5 days $\frac{1}{12}$ of it; for 7 days $\frac{1}{10}$ of it plus $\frac{1}{6}$ of the latter (6 days plus 1 day), etc.

170. Interest at Any Rate. If any other rate had been given in the problem in the above section, the process would be identical to the point of finding the interest at 6 % for the given time. If the rate were 5 %, the required interest would be $\frac{1}{6}$ less than \$72.24, or \$60.20; if 7 %, \$84.28, or $\frac{1}{6}$ more; if 4 %, \$48.16, or $\frac{1}{3}$ less; if 8 %, \$96.32, or $\frac{1}{3}$ more; if $4\frac{1}{2}$ %, \$54.18, or $\frac{1}{4}$ less, etc.

Hence to find interest at any rate: *Find the total interest at 6 % and increase or decrease it by the fractional part that the required rate is greater or less than 6 %.*

171. To find the Face of a Note.

PROBLEM. — The interest on a note for 8 mo. at 5 % was \$40. Find the face of the note.

\$40, interest at 5 % for 8 mo.
8, interest at 1 % for 8 mo. ($\frac{1}{5}$ of \$40).
\$48, interest at 6 % for 8 mo.
\$12, interest at 6 % for 2 mo. ($\frac{1}{4}$ of \$48).

Then, \$1200 (\$12 × 100) is the face of the note.

When the interest is given at any rate, *first change it to what it would amount to at 6 %.* This being the interest at 6 % for the given time, *such a fraction of it should be taken as will give the interest for two months only.* Since this would represent as many cents as dollars loaned, 100 *times that amount would be the face.*

172. To find Note Period.

PROBLEM. — The interest on $680 at 4½ % was $20.40. How long did the note run?

$20.40, interest at 4½ %.
6.80, interest at 1½ % (⅓ of $20.40).
$27.20, interest at 6 %.
6.80, interest on $680 for 2 mo.
3.40, interest on $680 for 1 mo.
$27.20 ÷ $3.40 = 8, the length of note period in months.

The interest given is changed to the interest at 6 %. The interest on the given principal for 1 month at 6 % is then found, and divided into the interest for the whole term at 6 %. The quotient is the number of months the note runs.

173. To find the Rate of Interest.

PROBLEM. — If a note for $660 draws $38.50 in 1 yr. 2 mo., what is the rate of interest?

$660 for 1 yr. 2 mo. would earn $46.20 at 6 %, and ⅙ of that, or $7.70 at 1 %. The required rate is as many per cent as $7.70 is contained times in $38.50, or 5 %.

The interest at 1 % for the given time, divided into the given interest, will give the rate required.

174. To find what Sum will produce a Given Amount.

PROBLEM. — What will produce $3955.95 at 4 % for 1 yr. 6 mo. 18 da.?

$.01, interest $1 will produce in 2 mo. at 6 %.
.09, interest on $1 for 1 yr. 6 mo. (18 mo.).
.0025, interest on $1 for 15 da.
.0005, interest on $1 for 3 da.
.093, interest on $1 for 1 yr. 6 mo. 18 da.
.031, interest on $1 at 2 % (⅓ of $.093).
.062, interest on $1 for 1 yr. 6 mo. 18 da. at 4 %.

$1 will amount to $1.062 in 1 yr. 6 mo. 18 da. at 4 %.

$3955.95 ÷ $1.062 = 3725, or the number of dollars which will amount to $3955.95 at 4 % for the given time.

The amount of $1 *for the given time and rate is first found. The number of times that is contained in the given amount is the number of dollars required to produce that amount.*

175. Other Methods. Interest tables, with the interest already calculated for any number of dollars for any number of days at given rates, are largely used by banks, trust companies, insurance offices, etc. The calculated interest is arranged in tabulated form for convenient reference, and the work of computing interest is thereby greatly lessened. Many different arrangements of such tables are published.

The formal 6 % method is often used by accountants, particularly when the time is more than 1 year. It is based on finding the interest on $1 for the given time, and multiplying that by the number representing the dollars loaned. Thus, $1 could bear $.06 interest in one year, $.005 interest in one month, $.000$\frac{1}{6}$ interest for one day, etc.

The interest at any other rate would then be found in the same way as by the 60-day method. (Sec. 169.)

176. Partial Payments. Payments on long-term notes, made from time to time, are known as *partial payments.* These payments, with their dates, are usually indorsed on the back of the note.

By what is known as the *United States Rule*, each payment is deducted from the amount of the debt at the time of the payment, and the remainder or new *principal* will bear simple interest until the next payment. If a payment is not as large as the accrued interest, it cannot reduce the debt. It is therefore disregarded, and added to the next payment.

By the *Merchants' Rule*, the sum of all the partial payments, together with the interest on each of them from its date until maturity or date of settlement of the note, is sub-

tracted from the sum the original principal would amount to, at simple interest, on the date of settlement.

Because merchants and bankers prefer short-term notes, with renewal (new note) for part of debt remaining unpaid, long-term notes, except when in the form of mortgage notes or bonds, are disappearing from business. Partial payments on mortgage notes and bonds may sometimes be made, but only in stipulated amounts and at specified dates, while the interest is covered by coupon notes, which are treated as separate obligations. On account of its occasional use in some sections of the country, the following illustration of partial payments is given:

PROBLEM. —

$500.00 OMAHA, NEB., Jan. 1, 1920.

On demand, I promise to pay C. L. Webster, or order, Five Hundred and 00/100 Dollars, value received, with interest from date at 6 %. J. L. FERNBY.

If endorsed March 1, 1920, $105; July 1, 1920, $108; Jan. 1, 1922, $20. What is due at settlement, Nov. 1, 1922?

$500,	original debt.
5,	interest 2 mo. (Jan. 1 to March 1, 1920).
505,	due March 1, 1920.
105,	payment Jan. 1, 1920.
400,	unpaid debt or new principal.
8,	interest 4 mo. (March 1 to July 1, 1920).
408,	due July 1, 1920.
108,	payment July 1, 1920.
300,	unpaid debt July 1, 1920.
	$27, interest for 18 mo. (July 1, 1920 to Jan. 1, 1922).
	$20, payment Jan. 1, 1922, deferred.
42,	interest for 28 mo. (July 1, 1920 to Nov. 1, 1922).
342,	due Nov. 1, 1922.
20,	payment Jan. 1, 1922.
$322,	net amount due at settlement.

PROBLEMS

1. Mr. Williams has the use of $240 for 1 yr. at 6%. What amount of interest does he pay?

2. A merchant borrows $260 for 1 yr. at 5%. How much does he pay for its use?

3. At 4% what must I pay for the use of $600 for 1 yr.?

4. Mr. Constable borrowed $750 for 1 yr. at 8%. How much interest did he pay?

5. A note of $324 for 2 mo. at 6% will draw how much interest?

6. A banker loans $240 for 4 mo. at 6%. How much interest does he receive?

7. A farmer sold a horse, taking a note of $175 for 6 mo. at 6% in payment. Find amount due at the end of the time.

8. A manufacturer borrows $1250 for 3 mo. at 6%. How much interest does he pay?

9. B borrows $75 for 1 mo. at 6%. What amount is due at the end of the time?

Find Interest:

10. On $150 for 1 mo. @ 6%.

11. On $720 for 4 mo. @ 6%.

12. On $480 for 3 mo. @ 6%.

13. On $70 for 5 mo. @ 6%.

14. On $65 for 10 da. @ 6%.

15. On $120 for 45 da. @ 6%.

16. On $160 for 6 da. @ 6%.

17. On $142 for 3 da. @ 6%

18. On $85 for 6 mo. @ 8%.

19. On $460 for 8 mo. @ 5%.

20. On $540 for 10 mo. @ 7%.

21. On $60 for 2 yr. 6 mo. @ 6%.

22. On $54 for 1 yr. 2 mo. 15 da. @ 6%.

23. On $480 for 75 da. (60 + 15) @ 6%.

24. On $76 for 63 da. (60 + 3) @ 6%.

25. On $960 for 90 da. (60 + 30) @ 6%.

26. On \$24 for 85 da. (60 + 10 + 15) @ 6 %.

27. On \$36 for 123 da. (2 60's + 3) @ 6 %.

28. On \$48 for 54 da. (60 − 6) @ 6 %.

29. On \$54 for 36 da. (30 + 6) @ 6 %.

30. On \$64 for 42 da. (30 + 12) @ 6 %.

31. On \$480 for 18 da. (3 6's) @ 6 %.

32. On \$730 for 72 da. (60 + 12) @ 8 %.

33. On \$75.60 for 87 da. @ 8 %.

34. On \$1244 for 8 mo. 20 da. @ 7 %.

35. On \$450 for 1 mo. 20 da. @ 5 %; 4 %.

36. On \$860 for 3 mo. 15 da. @ 4 %; 8 %.

37. The interest on a note was \$40, the rate 6 %, and the time 8 mo. Find the face.

38. What was the face of a note, if the time was 5 mo., the interest \$16, and the rate 6 %?

39. The time was 7 mo., the rate 6 %, and the interest \$13.30. What was the face of the note?

40. I paid a bank \$3.44 interest. I had the money 8 mo. at 6 %. How much did I have?

41. The interest at 6 % on a note for 1 yr. 3 mo. was \$3.15. What was the face?

42. What was the face of a note given for 5 mo. at 6 %, if the interest was \$19.50?

43. The interest on \$680 at 6 % was \$27.20. How long did the note run?

44. How long did a note for \$800 at 6 % run, if the interest was \$38?

45. Mr. Wood paid \$20.90 interest at 6 % on a note of \$380. How long did he have it?

46. If the principal was \$120, rate 6 %, and interest \$7.80, what was the time?

47. A note of \$150 drew \$8 interest at 8 %. How long had it run?

48. The interest on a note of \$180 at 8 % was \$12. How long had it run?

49. If a note for \$650 draws an interest of \$39 in 1 yr., what is the rate?

50. If I pay $60 on $1000 for 1 yr., what is the rate?

51. At what rate would $710 produce $17.75 in 5 mo.?

52. The interest on $640 for 4 yr. was $179.20. What was the rate?

53. What rate was charged if a merchant paid $43½ for the use of $650 for 16 mo.?

54. A farmer deposited $425 in the bank and at the end of 8 mo. drew out $437.75. What rate of interest did he receive?

55. The amount due on a note of $120 at the end of 8 mo. was $124. What rate of interest was charged?

56. Principal $200, time 3 yr., interest $30. Rate?

57. Principal $180, rate 4%, interest $14.40. Time?

58. Time 8 mo., rate 6%, interest $7. Principal?

59. Principal $76, time 3 yr., amount $87.40. Rate?

60. Principal $138.40, rate 4%, time 2 yr. 6 mo. 15 da. Interest?

61. Amount $3961.26, rate 4%, time 1 yr. 6 mo. 18 da. Principal?

62. Amount $502.09, rate 6%, principal $460. Time?

63. Time 4 yr. 6 mo., rate 5%, amount $7181.80. Interest?

64. Principal $7548, time 3 mo. 5 da., interest $119.51. Rate?

65. Principal $900, rate 5%, interest $56.25. Time?

66. Principal $72, rate 6%, time 2 yr. 6 mo. Interest?

67. Principal $460, rate 4%, time 1 yr. 5 mo. 6 da. Interest?

68. Principal $126, rate 7½%, time 2 yr. 3 mo. 18 da. Amount?

69. Principal $640, rate 6%, interest $86.40. Time?

70. Rate 6%, time 1 yr. 6 mo. 24 da., interest $338.40. Principal?

71. Principal $3200, time 2 yr. 6 mo. 6 da., amount $3602.67. Rate?

72. Principal $324, rate 5%, time 1 yr. 4 mo. 15 da. Interest?

73. Principal $960, rate 5%, interest $114. Time?

74. Principal $370, time 1 yr. 6 mo. 15 da., interest $34.23. Rate?

75. Rate 8%, time 2 yr. 8 mo. 24 da., interest $125.73. Principal?

76. Principal $2125, time 11 mo. 24 da., amount $2250.38. Rate?

77. Principal $1440, rate 6%, amount $2100. Time?

78. A note for $450, dated April 15, 1914, payable on demand, with interest at 6%, bears the following indorsements: June 1, $50; Oct. 10, $100; Nov. 15, $35; Dec. 10, $75. What is due Feb. 12, 1915?

79. What is due May 15, 1916, on a note for $600, dated July 12, 1914, bearing 5% interest, with the following indorsements: Sept. 21, 1914, $75; Feb. 18, 1915, $200; May 1, 1916, $65?

80. A note for $1500, dated Nov. 14, 1915, and due Nov. 14, 1917, with interest, has the following indorsements: Jan. 27, 1916, $340; May 13, 1916, $250; Sept. 16, 1916, $25; Feb. 17, 1917, $20. What is due at maturity?

Compute exact interest on the following. (See Sec. 166 and Prob. 1, at close of Chapter IX.)

81. $720 for 73 da. at 8%.

82. $460 for 63 da. at 5%.

83. $860 for 1 mo. 20 da. at 8%.

84. $800 from Jan. 14, 1900 to June 18, 1900, at 8%.

85. $760 from May 16 to July 27, at 7%.

86. $154.80 from Jan. 15 to Feb. 16, at 5%.

87. $172.50 from Nov. 12, 1891 to July 6, 1892, at 4%.

88. $465.20 from Jan. 29, 1901 to Jan. 1, 1903, at 5%.

89. $900 from June 15, 1901 to Jan. 15, 1904, at 4½%.

90. $140 from May 1 to Sept. 25, at 8%.

91. $560 from Jan. 24 to Dec. 16, at 5%.

92. $37.50 from April 3 to May 1, at 8%.

93. $184.50 from June 4, 1901 to July 16, 1903, at 7%.

XVIII

BANKING AND DISCOUNT

177. Banking. A *bank* is a business institution for the receiving and safe-keeping of money, and the making of loans. It also deals in credits, discounts negotiable paper, collects accounts, and makes payments in other cities.

A bank is usually a corporation, chartered by law, and is subject to supervision by the federal or state authorities, as a means of safeguarding the interests of depositors.

178. National Banks. Banks organized under "The National Banking Act" are entitled to the use of the word "National" in their name. Besides transacting a general banking business, they may issue "bank notes," which are secured by the deposit of United States Government bonds with the United States Treasurer. The government imposes certain limitations as to the modes of doing business, the amount of the reserve, etc. It requires uniform reports, makes regular examinations into the bank's business, and prescribes penalties for violation of the requirements of the bank law.

179. State and Private Banks. State banks are organized under the provisions of the statutes of the various states, and are subject to such restrictions as those statutes impose. State banks do not issue circulating notes and are not required, therefore, to hold United States bonds. They are not usually so carefully and frequently examined, are not

required to make such rigid reports, and are not restricted in so many ways in the conduct of their business, as are the national banks.

Individuals or firms may operate private banks and transact a general banking business, *i.e.* receive deposits, make loans, deal in exchange and other commercial paper, etc. They are usually subject to statutory provisions designed to insure the safety of funds deposited.

180. Savings Banks. Savings banks are also organized under state laws. Receiving small as well as large deposits, and paying interest thereon, they are designed to promote economy and encourage thrift.

The money deposited begins to draw interest on the first day of the following month or of the following quarter, according to the rules of the particular bank. The days on which deposits begin to earn interest are known as *interest days*. When interest days are quarterly, they are January 1, April 1, July 1, and October 1.

Interest is usually allowed only on those sums which have been on deposit for the full time between interest days. Thus, the lowest daily balance in the month or quarter, not counting fractional parts of a dollar, is the amount upon which interest is computed. The interest is usually declared semiannually. If not paid, it is credited to the depositor's account and draws interest thereafter.

181. Savings Bank Accounts.

PROBLEM 1. — The dividends of interest at a savings bank are declared semiannually. A customer deposited June 5, $300; Aug. 10, $150; Oct. 1, $100; Dec. 10, $20. No withdrawals having been made, what was due Jan. 1, following, if interest be reckoned on the deposits from the first of each quarter at 4 %?

Dates	Deposits	Daily Balances	Interest Days	Smallest Quarterly Balances	Quarterly Interest
June 5	$300	$300	July 1		
Aug. 10	150	450			
Oct. 1	100	550	Oct. 1	$300	$3.00
Dec. 10	20	570			
Jan. 1		570	Jan. 1	550	5.50
					8.50
					570.00
					$578.50

SOLUTION. — Since interest begins on the first of each quarter, the deposit of Aug. 10 will not begin to draw interest until the beginning of the second quarter. The interest on $300 for one quarter is $3. The deposits of Aug. 10 and Oct. 1, together with the balance for the first quarter ($550), will draw interest for the second quarter. The deposit of Dec. 10 will not draw interest until the beginning of the third quarter.

If interest days are monthly, the above account would balance as follows:

Dates	Deposits	Interest Days	Smallest Monthly Balances	Monthly Interest
June 5	$300	July 1	$300	
Aug. 10	150	Aug. 1	300	$1.50
Oct. 1	100	Sept. 1		1.50
Dec. 10	20	Oct. 1	450	2.25
		Nov. 1	550	2.75
		Dec. 1	550	2.75
		Jan. 1	570	2.75
				13.50
				4.50
				9.00
				570.00
				$579.00

The aggregate interest on the smallest monthly balances at 6% is found to be $13.50. At 4% it is $9. $570 + $9 = $579, the balance due Jan. 1.

PROBLEM 2. — What is the balance due July 1, on the following account? Deposits: Nov. 20, $300; Jan. 14, $200; June 10, $150. Withdrawals: March 20, $150; June 20, $100. Interest is declared every 6 months, and 3% per annum is allowed from the first of each quarter.

DATES	DEPOSITS	WITHDRAWALS	DAILY BALANCES	INTEREST DAYS	SMALLEST QUARTERLY BALANCES	QUARTERLY INTEREST
Nov. 20	$300		$300	Jan. 1		
Jan. 14	200		500			
Mar. 20		$150	350			
April 1			350	April 1	$350	$5.25
June 10	150		500			
June 20		100	400	July 1	400	6.00
						11.25
						5.63
						5.62
						400.00
						$405.62

The smallest balance for the first quarter is $350, and for the second quarter, $400. The quarterly interest on these balances aggregates $5.62. $405.62 is the balance due July 1.

PROBLEMS

1. A. B. Wilson made the following deposits in a savings bank: Dec. 20, $60; June 30, $80; Sept. 1, $100; Oct. 15, $200; Nov. 5, $100; Dec. 25, $200. The interest term is 6 months, and interest is allowed on balances from the first of each quarter at 4% per annum. What is the balance due Jan. 1?

NOTE. — The interest is compounded at end of each interest term.

2. A. G. Thomas made the following deposits in a savings bank: Dec. 15, $300; Jan. 14, $300; Feb. 25, $150; June 5, $100; July 1, $120. The interest term is 3 months, and interest at the rate of 3% is computed from the first day of each quarter. What amount is due July 1?

3. A. M. Peabody deposits in a savings bank as follows: Jan. 1, $400; Feb. 20, $200; March 10, $150; April 10, $60; May 15, $5.50. He withdrew during the same time, as follows: Jan. 15, $100; Feb. 5, $150; April 20, $80; June 30, $120. The rate of interest is 4% per annum, the interest term 6 months, and the interest is computed from the first of each quarter. Find the amount due July 1.

4. A customer makes deposits in a savings bank as follows: Nov. 25, $600; Jan. 1, $200; May 20, $50; June 25, $100. If the interest term is 6 months, and interest is computed from the first day of each month at 4%, what is due July 1?

NEGOTIABLE PAPER

182. Checks. Money on deposit in a bank in an "open account" is subject to check. That is, the bank will pay out any part or all of it upon a written order from the depositor. This order is called a *check*. Checks are usually made payable to a certain person known as the *payee*, or to his order, although sometimes they are made payable to the bearer. When the check is paid by the bank, it is stamped "paid" and is finally returned to the depositor who signed the check, known as the *maker*. To withdraw money from deposit, the check is made payable to "self."

If there are no funds on deposit, a check is worthless. Obtaining money from a third party on such a check is against the law.

For certain purposes, it is desirable to have a check show on its face that it represents actual value. To do this, the maker or payee takes it to the bank upon which it is drawn, and the paying teller or cashier of the bank writes across its face, and over his signature, that it is "good when properly indorsed." This makes of it a *certified check*. Enough

money belonging to the maker is set aside by the bank to pay it when presented. This amount will be paid, or released for regular account, only upon presentation of the check itself.

Checks signed by individual depositors of a bank are known as *individual checks*. Those signed by the cashier of a bank are known as *cashier's checks*. The latter are used in paying bank expenses, or to pay the proceeds of a note purchased, and are sometimes issued to customers to be used instead of drafts.

183. Certificates of Deposit. Money may be deposited for special purposes, in a "closed account," *i.e.* not subject to check. The bank issues a certificate of deposit, therefore, certifying that a certain sum is on deposit, which will be paid to the holder of the certificate when properly indorsed. This may be either a *demand* or a *time* certificate. Time certificates usually draw interest.

184. Notes. When promissory notes (Sec. 161) are made payable to a particular person, without having either of the two clauses, "or order" or "or bearer," they are payable to no person other than the payee, and are therefore not transferable. Such notes are *non-negotiable*. If either of the clauses are embodied, the note may be bought and sold, and it is, therefore, *negotiable*. Checks, notes, drafts, certificates of deposit, or any other papers representing value, which permit of being bought and sold, are likewise termed *negotiable papers*.

185. Exchange. *Bank Drafts.* The use of bank drafts as a means of exchange has already been treated in Sec. 118. As there stated, a bank usually maintains credit with another bank, its correspondent, in some of the larger cities and sells drafts on that credit for a certain fixed charge for

small amounts, and at so much per hundred dollars for larger amounts.

Sometimes the amount charged for exchange is expressed in per cents, and when so, it is fixed variously at between $\frac{1}{10}\%$ and $\frac{1}{4}\%$ of the amount sold. The price would depend upon the size of the draft and sometimes upon the demand for exchange on the particular city in which the paying bank is located. Formerly it was the custom to quote exchange at either a premium or a discount, dependent upon whether the amount of credit on a given city was greater or less than the demands of business. Of late years, this finds very little use, and drafts are sold at par, the only charge being that for the service.

186. Collection by Draft. *Individual drafts* form a very common means of collecting delinquent accounts. If a debtor has paid no attention to repeated statements or letters, he is notified that unless heard from by a certain date, he will be drawn upon for the amount of the debt.

If no word comes, a draft is drawn on him in favor of the creditor, and made payable "at sight," or some number of days after sight. This is usually deposited in the bank with which the creditor does business, and it is sent to a bank in the city of the debtor's residence indorsed "for collection," and the latter bank will present it and endeavor to have it paid. If successful, the bank will remit the amount, less a small fee for the service. If the draft be not paid, it will be returned with a memorandum giving the reason, should one be given by the debtor, for its nonpayment. Repeated refusal or failure to pay such drafts leaves one open to suspicion of unreliability.

If the draft is for a period of time after sight, the drawee who wishes to honor the draft writes the word *accepted*, with date and signature, across the face of the draft, which

thereupon to all intents and purposes becomes a promissory note, and the time period is counted from the date of the *acceptance*. If a draft reads "after date," then the time period begins with the date of the draft.

The signer of a draft is the *drawer*, the person or bank on whom it is drawn is the *drawee*, and the person to whom it is to be paid is the *payee*.

In much the same way shippers may use individual drafts to secure payment on a shipment of goods to an unknown or unreliable customer. The goods are billed to the shipper at the address of the customer, and the bill of lading, together with the draft for the amount of the invoice, is sent to a bank for collection. Upon payment of the draft, the customer is given the bill of lading, which entitles him to the delivery of the goods shipped.

187. Indorsement. When one wishes to cash a check, he writes his name on the back of the check and presents it for payment. This is known as an *indorsement*, and its effect is to transfer ownership of the check, or the money it represents, to whoever may hold the check.

Blank indorsement
A. M. Hawkins

Particular indorsement
Pay to Cutter Lumber Co.
James Morton

When one wishes to sell a negotiable note, the payee, likewise, indorses it upon the back. This may be done "in blank" by simply writing his name, when it becomes payable to the bearer of the note; or it may be indorsed "in particular" by writing "pay to —— ——" and signing it, when it is payable only to the one whose name is written; or it may be indorsed "in full," by writing "pay to the order of —— ——" and signing it. This latter in-

Full indorsement
Pay to the order of
Howard & Sons
G. W. Dawson

dorsement transfers *full* rights of the payee to the purchaser of the note. Drafts may be indorsed in the same way as notes with exactly the same effect.

Indorsement of a check, note, or draft makes the indorser responsible to the indorsee for the payment of the paper in case the maker fails to pay, unless it is indorsed "without recourse." When there is more than one indorser, the order of responsibility follows the order of indorsement.

BANK DISCOUNT

188. Discounting Negotiable Paper. Individual drafts and notes have, or may have, a time period. If so, they are not worth their full value until they are due.

To realize on negotiable paper of this kind before it is due, it is sold. The price received is necessarily less than its value at maturity, and the amount deducted is called the *discount.* The note or draft is thus *discounted*, and the amount of discount represents the interest upon the money advanced by the purchaser.

Likewise, money may be borrowed by giving a note to a bank, payable in a given period of time, and discounting it.

189. Maturity Value. If the paper discounted does not bear interest, the face value is its value at maturity. If it is interest-bearing from date, the interest which will have been earned at date of maturity must be added to the face of the note or draft to find its *maturity value.* Since it is necessary for the purchaser to know what the value of the paper will be when due, before he can wisely decide what or how much to pay for it, to find the maturity value is the first step in calculating discount.

190. Term of Discount. The length of time to elapse between the date of purchase of negotiable paper and the date it is due is called the *term of discount.*

In calculating the term of discount, banks count the exact number of days the paper is yet to run. If the state law permits days of grace, they are counted in the term of discount. If the paper becomes due a certain number of *months* after date, it will mature on the same day of the month as the date of the note. If the paper matures a certain number of *days* after date, the exact number of days is counted to determine the date of maturity. If the time expires on Sunday or on a legal holiday, it matures the day after. Some banks count both the day of purchase and the day of maturity in determining the term of discount.

191. Term of Discount Table. For convenience, the following table is often used by bankers to find the days intervening between dates:

From any Day of	To the Same Day of the Next											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	33	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	122	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of one month to the same day of another month, within a year, is found by starting at the name of the first month (in the left-hand column), and following across to the column of the last

month directly opposite. Thus, to find the number of days from June 10 to Nov. 15, we note that it is 153 da. from June 10 to Nov. 10, and add 5 da. (Nov. 10 to Nov. 15), which gives 158 da. as the exact time.

192. Discount is Interest. Simple interest upon the maturity value of the paper discounted, for the length of time indicated by the term of discount, forms the amount of discount. This interest or discount is deducted, and the remainder of the maturity value is paid the owner for the paper. This remainder is called the *proceeds*.

193. True Discount. Because bank discount is interest, computed on the maturity value and is always collected in advance, interest is charged upon the amount of the discount itself, although the latter is never received by the one who is selling the note. This fact permits a smaller rate of interest in discounting, and the matter is thus equalized.

There is a form of discount, however, which asks interest only on the money actually advanced. This is known as *true discount*. The money advanced is the *present worth*. It is the sum of money which, at interest for the discount term, will amount to the maturity value of the note. The difference between the present worth and the maturity value is the "true discount." Thus, \$1 at interest 4 months at 6% would amount to \$1.02. It would take as many dollars to amount to \$1224 as \$1.02 is contained in \$1224, or \$1200. This is the present worth of a note for \$1224 discounted 4 months before it was due at 6%. The "true discount" is the difference between those amounts, or \$24.

True discount has little or no place in dealing with negotiable paper. It is sometimes of value in personal calculations connected with business ventures. No problems are here given for true discount, but the exercises in bank discount may also be used for practice in true discount.

194. To find the Bank Discount and Proceeds.

PROBLEM 1. — A note for $800, without interest, was discounted 60 days before it was due. What were the proceeds?

$800, maturity value of note.
8, interest (discount) at 6% for 60 days.
$792, proceeds of discounted note.

PROBLEM 2. — What would be the proceeds of a note for $640, running 90 days from Sept. 1, 1904 with interest, and discounted Oct. 1, 1904, at 6%?

$640.00, face value.
9.60, interest for 90 days.
$649.60, maturity value of note.
6.50, interest (discount) 60 days (Oct. 1 to Nov. 30).
$643.10, proceeds of note Oct. 1.

195. To find the Face.

PROBLEM. — For what amount must a note be drawn for 90 days so that it will give $1000, when discounted for the full term at 6%?

$1.00, face of note for $1.
.015, interest (discount) on $1 for 90 days at 6%.
$ .985, proceeds of $1, discounted for 90 days.
$1000 ÷ $.985 = $1015.23, face of note.

If a note for $1 when discounted at 6% for 90 days will produce $.985, the note required to produce $1000 would have to be for as many dollars as $.985 is contained times in $1000, or for $1015.23.

EXERCISES

By inspection, find the date of maturity of each of the following notes:

	Date	Time		Date	Time
1.	May 28, 1905	1 mo.	3.	April 24, 1906	90 da.
2.	June 6, 1904	1 mo.	4.	Feb. 28, 1906	60 da.

Find the date of maturity of the following drafts:

	Date Accepted	Time		Date Accepted	Time
5.	May 9	20 da.	**7.**	Jan. 30	30 da.
6.	June 12	1 mo.	**8.**	Sept. 6	3 mo.

Find date of maturity and term of discount:

	Date of Note	Time	Date of Discount
9.	May 15	3 mo.	June 1
10.	Sept. 20	30 da.	Sept. 29
11.	March 18	2 mo.	March 31

	Date of Draft	Time after Sight	When Accepted	When Discounted
12.	Feb. 16	10 da.	Feb. 17	Feb. 18
13.	March 20	30 da.	March 22	March 25
14.	July 1	60 da.	July 5	July 20

	Date of Draft	Time after Date	When Accepted	When Discounted
15.	July 6	3 mo.	July 10	Aug. 4
16.	Aug. 8	1 mo.	Aug. 9	Aug. 9
17.	Sept. 15	30 da.	Sept. 15	Sept. 18

PROBLEMS

Find bank discount and proceeds:

	Face	Date of Note	Time	Date of Discount	Rate of Discount
1.	$1200	June 2	30 da.	June 6	6%
2.	$2500	April 8	60 da.	April 20	6%
3.	$3000	Aug. 10	2 mo.	Aug. 15	5%
4.	$ 600	Sept. 16	2 mo.	Sept. 20	8%
5.	$7200	Jan. 14	90 da.	Jan. 25	4½%
6.	$3500	May 20	30 da.	May 21	7%

7. May 15, A. B. Kittridge & Co. borrowed of the First National Bank $1600 on their note at 60 days. Find the proceeds, the rate being 7%.

8. I borrowed $450 of the Atlas Bank on my note for 70 days. Write the note, and find the proceeds, the rate of discount being 6%.

Find the date of maturity, the term of discount, the discount, and the proceeds of the following notes and drafts:

9. $1800.00 COLUMBUS, O., April 12.

Sixty days after date I promise to pay to the order of O. E. Chase & Sons, Eighteen Hundred Dollars, at the Commercial National Bank. Value received.

Discounted May 8, at 5%. A. B. GRINDLE.

10. $660.00 MINNEAPOLIS, MINN., Sept. 1.

Three months after date I promise to pay to the order of Freeman & Comstock, Six Hundred Sixty Dollars, at the Bank of Commerce. Value received.

Discounted Oct. 10, at 4½%. C. M. DUNLAP.

11. $850.00 CLEVELAND, O., May 1, 19—.

Six months after date I promise to pay to the order of A. Douglas & Co., Eight Hundred Fifty Dollars, with interest at 5%. Value received.

Discounted June 6, at 6%. C. L. TRUEBLOOD.

12. $875.00 LEAD, S. DAK., May 29, 19—.

Eight months after date I promise to pay to the order of L. M. Gittinger, Eight Hundred Seventy-five Dollars, with interest at 6%. Value received.

Discounted June 25, at 7%. OLIVER C. DITSON.

13. $2750.00 KANSAS CITY, MO., March 13, 19—.

At sixty days' sight pay to the order of ourselves _____

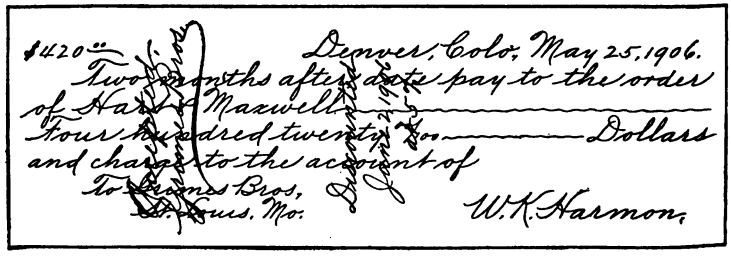
Twenty-seven Hundred Fifty $\frac{00}{100}$ _____ Dollars.

Value received, and charge the same to the account of

To ENSIGN BROS., G. W. PATCHELL & Co.
SANDUSKY, O.

Accepted March 30. Discounted April 4, at 7%.

14.



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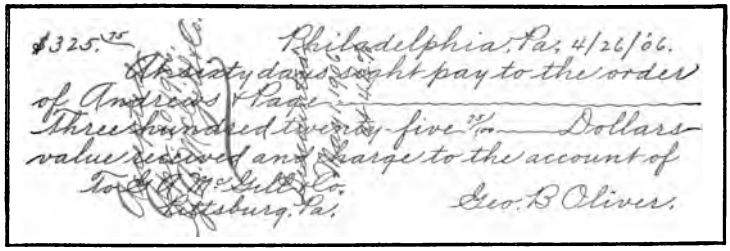


16. $675.50 CINCINNATI, O., Dec. 23, 19—.

Ninety days after date pay to the order of ourselves, Six Hundred Seventy-five and $\frac{50}{100}$ Dollars. Value received, and charge to the account of

To A. B. HIMES, INDIANAPOLIS, IND. ANTWERP & BRAGG.

Accepted Jan. 2. Discounted Jan. 4, at 7%. Collection charges $\frac{1}{10}$%.

NOTE. — Collection is charged on the face.

17. I wish to borrow $600 at the bank. For what sum must I issue a 60-day note to obtain the amount, discount being 6%?

18. I owe $960, and have my note discounted at the bank for 75 days at 6% for such a sum that the proceeds will pay the debt. What was the face of the note?

19. A merchant purchased goods for $875 on 3 months' credit, 5% being offered him for cash. He accepted the cash offer and borrowed the money at the bank, giving his note for 60 days at 6%. What was the face of the note, and what did he gain or lose by so doing?

20. A bank draft for $7500 was bought for $7496.25. What was the rate of exchange? At the same rate, what would be the cost of a draft for $14,500? one for $125,455.60? one for $12,367.50?

21. An agent sold a carload of 26 cattle, averaging 1125 lb., at $5.60 per hundred weight. He paid $135 freight, $26.75 for feed, and charged 2% commission for selling. He buys a draft at $\frac{1}{8}$% premium with the proceeds. What is the face of the draft?

22. Snyder & Co. of New Orleans drew a draft on A. M. Hawkins of Boston, Mass., for $7865.50, which they sold at the bank at $\frac{3}{8}$% discount. What were the proceeds?

23. Complete the following letter of advice. The rate of collection on the first two items is $\frac{1}{10}\%$, and on the others $\frac{1}{8}\%$.

SECOND NATIONAL BANK

RICHMOND, IND., June 20, 1917.

MR. WM. J. DOYLE, CASHIER,
Atlas Bank,
Cincinnati, Ohio.

DEAR SIR:

We credit your account this day for the proceeds of collections as stated below.

Respectfully yours,

JAMES W. KING, CASHIER.

YOUR NO.	PAYER	AMOUNT		CHARGES		PROCEEDS	
620	A. M. Pierson	600	00				
415	F. T. Davis	560	75				
930	Murphy, Grant & Co.	3545	10				
560	Richmond Chemical Co.	12345	80				
748	S. F. Carroll	800	00				

XIX

STOCKS AND BONDS

196. Organizations for Business. If an individual engages in business by himself, he is entitled to all the profits and assumes personal liability for all debts. Should it be desirable to have the capital or services of more than one person, a *partnership* is formed. The business is then done under a *firm* name, *e.g.* Merritt & Saunders, John N. Baldwin & Co., Parlin, Orendorff & Co., etc.

There is usually a written agreement for such a partnership, specifying the services, money, or property contributed by each to the business, and stating what part of the profits each is to receive. Each partner may bind the firm by his acts, and each is personally liable for the firm's indebtedness.

197. Corporations. Whenever large capital is needed for a business, or investors wish to limit their financial responsibility to the amount invested, or when the range of the business is wide, or individuals wish to invest in, but not to give their personal attention to a business, and for various other reasons, a *stock company* or *corporation* is organized.

Such an organization is usually created under, and must conform to, state laws. Its affairs are conducted by officers, selected in a prescribed way. When organized it becomes before the law a *body corporate*, vested with the same rights as an individual; to have and to hold property, to contract debts (within the limits of the law), to sue and be sued, etc.

198. Articles of Incorporation. In order to form a corporation, the investors or *incorporators* sign, file in some government office, and publish their *Articles of Incorporation.* These articles set forth the purposes of the organization, the amount of money subscribed by each, the *capital stock*, the total amount that may be subscribed or *authorized capital*, the number of *shares* or parts into which the capital is to be divided, the face or *par value* of each share, the name of the company, its place of business, its officers, etc.

199. Certificates of Stock. When the legal requirements are satisfied, the subscribers pay into the treasury the stipulated price and receive a *certificate of stock*, stating the number of shares bought, the par value of each share, etc. The owner of such certificate is entitled to a part in the control of the business corporation, and to participate in its property and its profits in proportion to the number of shares owned.

200. Dividends and Assessments. At stated periods, the condition of the business is ascertained. If profits are shown, a portion is distributed among the stockholders as a *dividend.* Gains remaining are termed *undivided profits.*

If a loss is shown, which the necessities of the business require should be made good, it is apportioned among the stockholders as an *assessment* to be paid by them. Dividends and assessments are usually expressed in per cents of the face value of the stock.

202. Kinds of Stock. Corporations often issue both *preferred* and *common stock.* Dividends up to a certain limit, usually 5 % to 7 %, are first paid on preferred stock. Profits remaining may then be divided on common stock. Holders of preferred stock thus have first chance for dividends, but their dividends are limited. Law or charter also sometimes limits dividends on common stock.

202. Premium and Discount. When dividends declared by a given concern are higher than the prevailing rates of interest, the stock of such a company will naturally sell for more than the face value of the shares. It is then said to be *above par*, or at a *premium*. Should the dividends be less than current interest rates, the stock will not bring its face value and is said to be *below par*, or at a *discount*.

The market value of stock is the amount it will bring in the market, and is usually stated as a per cent of the par value. Thus, stock quoted at 26, 84, or 120 means that a hundred dollars in stock of a company is worth $26, $84, or $120, respectively.

203. Liability of Stockholders. In general, a holder of stock in a business corporation is liable for the debts of the corporation only to the extent of the par value of the stock. The National Banking Law, however, makes stockholders in national banks liable to the extent of the par value in addition to what they have paid for the stock.

Some stock is, by the terms of the charter or the by-laws, *non-assessable*. In such companies, the entire risk assumed by the holder of the stock is the amount paid for it.

204. Bonds of Corporations. Corporations may borrow money, pledging their property as security, in the same way as individuals. If some particular property is pledged, it is upon an ordinary promissory note, with real or chattel mortgage.

When the amount of money to be raised is large, it is done by formally issuing *bonds*, and placing them upon the market for sale. A bond, then, is a mortgage note, upon which the corporation pays interest, and to the payment of which the entire property and business of the corporation is pledged.

205. Government Bonds. National and state governments, counties, townships, cities, school districts, etc., are by law declared to be "bodies corporate." As such they may issue bonds, within certain limits, and agree to pay a limited rate of interest. For the redemption of such bonds a tax is levied to create a *sinking fund* for their payment when due. In default of payment, the courts may issue judgment and cause a special tax to be levied and collected for their payment.

206. Kinds of Bonds. When bonds are made payable to the owner or his assignee, they are termed *registered bonds*. The names of owners are registered, and the interest is sent directly to them.

When the bonds are made payable to bearer, the interest is provided for in attached notes, termed *coupons*. A coupon is surrendered when each interest payment is made. Bonds of this sort are termed *coupon bonds*.

207. Bond Values. The value of a business corporation bond depends upon the amount of property or business of the company. If the property is large or the business prosperous, then the bonds are reliable. Their market value also depends upon the rate of interest the bonds bear. If it is higher than the current interest rates, the bonds form a profitable investment, and will, therefore, tend to sell at a *premium*. If the interest rate is less than current rates, or if the bonds are not absolutely secure, they will probably sell at a *discount*.

208. Stock Quotations. The price of stocks or bonds is usually quoted at a certain per cent of their par value. So many elements enter into the question of the value of stocks and bonds in the open market, and so easily is the confidence of the investing public weakened or strengthened, that

the market quotations of stocks or bonds often fluctuate widely.

The necessity for an intimate knowledge of corporation and market conditions gives rise to brokerage firms. These *brokers* advise their clients, buy and sell stocks, bonds, etc., and charge a small *commission* or *brokerage*, usually about $\frac{1}{8}$ of one per cent, for the service. Brokerage is always a per cent of the par value, whether for buying or for selling.

209. Stock Exchanges. So important are stocks and bonds that special organizations of dealers are formed, known as stock exchanges, boards of trade, etc. At these exchanges, brokers buy and sell for investors. If for speculation, investors usually buy because of an expected rise in the market value, expecting to sell at an advanced price. They do not always pay the full price of the stock, but a part only, leaving the certificate of stock with the broker as security for the remainder. Thus, stock may be bought upon a 20% margin, by paying 20% of its value, depending on profits to pay the remainder or using the amount paid to cover losses if the stock goes down.

There is usually a group of *operators*, who are interested in forcing the price of certain stocks upward. These are known as *bulls*. There are others who wish to see the price lowered, and these are called *bears*. Bears may either wish to purchase good stock at a cheap price, or in the belief that the stock was certain to go down, they may have sold largely of the stock without owning it, and wish, therefore, to force it down so they may purchase cheaply what they have bargained to deliver. In the latter case they are said to have sold *short*.

210. Market Quotations. The following quotations show the highest and the lowest market quotations for a given year as furnished by Bradstreet's Commercial Agency:

STOCKS

	High	Low
Adams Express	250	236
Amalgamated Copper	91	70
American Beet Sugar	34½	23
American Express	246	209½
Atchison, Topeka & Santa Fé	93⅜	71½
Brooklyn Rapid Transit	91½	56⅞
Chicago, Burlington & Quincy	250	201
Chicago, Milwaukee & St. Paul	187½	168½
Consolidated Coal	73	24½
Erie	52¼	37½
General Chemical	72¾	37½
Illinois Central	183	152¼
National Biscuit	66⅜	52
National Biscuit Pfd.	120¼	110
National Lead	77½	24½
Pittsburg, Ft. Wayne & Chicago	185	182½
Pressed Steel Car	53⅜	34
Quicksilver	1⅞	¾
Rubber Goods Mfg. Co.	39	25
Union Pacific	138⅜	113
United States Leather	16	10⅞
United States Steel	39½	24⅞

BONDS

	High	Low
Am. Hide & Leather 6's	97½	97½
Chesapeake & Ohio 6's, 1911	110	110
Denver & Rio Grande 4's	100⅞	97½
Illinois Central 4's, 1952	108	107½
Missouri Pacific 4's	96	95
Seaboard Air Line 5's	104½	104
United States reg. 4's, 1907	104½	104½
Pennsylvania 4½'s	109	108½
United States coupon 4's	104¼	104

211. Illustrative Problems. In all problems in this text, the par value of a share of stock will be taken as $100, unless otherwise stated.

1. A broker sells 145 shares of stock at $125\frac{3}{8}$; brokerage $\frac{1}{8}\%$. What should his principal receive?

$125\frac{3}{8}\% - \frac{1}{8}\% = 125\frac{1}{4}\%$, proceeds of each share.
$\$100 \times 145 = \$14{,}500$, par value of 145 shares.
$\$14{,}500 \times 1.25\frac{1}{4} = \$18{,}161.25$, proceeds.

2. A broker sold Union Gas stock for $\$26{,}250$ at 75% premium. How many shares did he sell? What was the par value of the stock?

$75\% = \frac{3}{4}$.
$\frac{7}{4}$ = selling price.
$\frac{7}{4}$ of par value $= \$26{,}250$.
$\frac{1}{4} = \frac{1}{7}$ of $\$26{,}250$, or $\$3750$.
$\frac{4}{4} = \$3750 \times 4$, or $\$15{,}000$, par value of stock.
$\$15{,}000 \div \$100 = 150$, number of shares.

3. What sum must be invested in 6% bonds at 120 to yield an annual income of $\$2820$?

6% of face value of bonds bought $= \$2820$.
$100\% = \frac{100}{6}$ of $\$2820$ or $\$47{,}000$ face value.
$120\% = \frac{6}{5}$.
$\frac{6}{5}$ of $\$47{,}000 = \$56{,}400$, investment.

4. What per cent profit does an investor make on stock that pays a dividend of 6%, if he buys at 75?

$\$6$ = income on one share.
$\$75$ = cost of that share.
$\frac{6}{75} = 8\%$, profit.

5. A year's net profits of the Plymouth Milling Co. were $\$15{,}275.50$. The capitalization of the concern is $\$200{,}000$, divided into 2000 shares. A dividend of $6\frac{1}{2}\%$ was declared, and the remainder of the profits was carried to surplus fund. Find the amount of dividend and the amount carried to sur-

plus fund. What amount will a man receive who owns 60 shares of stock?

$6\frac{1}{2}\%$ of $\$200{,}000 = \$13{,}000$, the dividend declared.
$\$15{,}675.50 - \$13{,}000 = \$2675.50$, surplus fund.
$6\frac{1}{2}\%$ of $\$6000 = \$390$, dividend on 60 shares of stock.

6. A manufacturing company is capitalized at $200,000. The gross earnings for a year are $25,185, and the expenses are $6785.50. After setting aside 2% for surplus fund, what even per cent of dividend may be declared?

$\$25{,}185 - \$6785.50 = \$18{,}399.50$, net earnings.
2% of $\$18{,}399.50 = \$367.99$, amount for surplus fund.
$\$18{,}399.50 - \$367.50 = \$18{,}031.50$, amount to be divided.
1% of $\$200{,}000 = \$2000$, amount of 1% dividend.
$\$18{,}031.51 \div \$2000 = 9$, the rate per cent of dividend, with $31.51 additional, carried to undivided profits.

PROBLEMS

Find the market value at the highest and lowest price of the following stocks and bonds, by use of the above market quotations:

1. 75 shares Adams Express.
2. 68 shares Erie.
3. 155 shares National Lead.
4. 85 shares Quicksilver.
5. 150 shares Union Pacific.
6. 7 Pennsylvania $4\frac{1}{2}$'s (Denom. $1000).
7. 9 Am. Hide and Leather 6's (Denom. $1000).
8. 245 shares National Biscuit pfd.

9. A man who holds 170 shares of stock receives a dividend of $1275. What was the rate of dividend declared?

10. An assessment of $306 is made on 72 shares of mining stock. What is the rate of assessment?

11. A broker sold for me 360 shares of gas stock at 145; brokerage $\frac{1}{4}\%$. What sum should I receive?

12. A railroad declares a dividend of 5%. How many shares does a man hold who receives a dividend of $435, if the par value of stock is $100?

13. What sum must be sent a broker that he may buy 150 shares of gas stock at 105; brokerage $\frac{1}{8}\%$?

14. A stockholder meets an assessment of \$167.50, which is levied at $2\frac{1}{2}\%$ on his stock. How many shares has he?

15. How many shares of Atchison, Topeka, and Santa Fé stock can be bought for \$8100 at $89\frac{1}{2}$; brokerage $\frac{1}{8}\%$? What will the dividend on this stock amount to at 5%? What rate of interest would this be on the investment?

16. What amount of stock must be sold at $41\frac{1}{2}$ to yield \$8275, if brokerage is $\frac{1}{8}\%$?

17. How many shares of American Sugar at $31\frac{1}{2}$ can be bought for \$202,400; brokerage $\frac{1}{8}\%$? If this stock pays a dividend of $2\frac{1}{2}\%$, what rate of interest will a man receive on his investment?

18. What sum must be invested at 93 to bring an income of \$4800, if the rate of dividend is 4%?

19. What is the quotation of 7% stock that brings an income of 10%?

20. What rate per cent is realized on an investment by investing in 5% stock at 80?

21. A banker sold through a broker 150 shares of stock at $124\frac{1}{4}$, paying $\frac{1}{8}\%$ brokerage. What amount did each receive?

22. A speculator bought 2500 shares of the United States Steel stock at $33\frac{1}{4}$ and sold it at $39\frac{1}{4}$. What was his net profit after allowing $\frac{1}{8}\%$ brokerage each for buying and selling?

23. A man buys 120 shares of stock at $76\frac{1}{4}$ and six months later sells it for 85. In that time he received a dividend of $1\frac{3}{4}\%$. If money is worth 6% interest and he paid $\frac{1}{8}\%$ brokerage for buying and for selling, did he gain or lose, and how much?

24. The capital stock of a company is \$200,000. $\frac{1}{4}$ of this is preferred stock, entitled to 6% dividend. What rate of dividend is paid on common stock, if \$9000 is distributed in dividends?

25. An investor buys 604 shares of stock, par value \$50, for \$35 a share; brokerage $\frac{1}{8}\%$. Six months later he sells for \$58 a share. In the meantime he had received a dividend of 5%. Money being worth 6%, what did he gain or lose?

26. A bank with capital stock of \$150,000, declares a semi-annual dividend of $4\frac{1}{2}\%$. What is the amount of the dividend, and how much will a man receive annually who owns 275 shares?

27. A corporation has a capital stock of $100,000. Its net earnings for the year are $14,256.32. 4% of the net earnings is set aside as a surplus fund to cover losses, 7% dividend is declared, and the remainder is carried to undivided profits. What are the amounts carried to surplus fund, undivided profits, and to dividend accounts?

28. A merchant sold his business for $245,000. He invested $196,000 in Pullman stock at 195½, and the remainder in Erie preferred at 69¾. Pullman stock pays 12% and Erie 6% dividend; brokerage ⅛% in each case for buying. What was his annual income?

29. A broker purchased 500 shares Amalgamated Copper at 70; 250 shares C. B. & Q. at 201; 400 shares National Biscuit Co. at 66⅛; 475 shares General Chemical at 72¼, and 150 shares Western Union at 113. What is the total cost to his principal, if brokerage is ⅛%?

30. A speculator purchased 200 shares United States Steel at 24⅞; 600 shares Pressed Steel Car at 34; 700 shares Western Union Telegraph at 95½. He sold the Steel stock at 39½, the Pressed Steel Car at 53⅜, and the Western Union at 92, brokerage being ⅛% for buying and for selling. What was the net gain or loss?

31. A certain county, on Jan. 1, 1907, issued $250,000 worth of 5% 10-year coupon bonds. If these bonds were sold through a broker at 102½, how much was received by the county? Brokerage ¼%. If the interest is payable semi-annually, what is the amount of each interest coupon? How much must be levied in taxes each year to pay the interest and provide a sinking fund sufficient to pay the bonds in full at maturity? What would be the annual rate to be levied, if the assessed valuation of the county averages $46,875,000?

32. A broker bought for a customer 800 shares of United States Steel common, at a total cost of $30,100; brokerage ⅛%. Find market quotation of stock.

33. How much must be paid, including brokerage at ¼%, for a sufficient number of United States 4's at 123½ to obtain an annual income of $1200?

34. My broker, after selling 500 shares of Philadelphia Gas stock and deducting the usual commission, remitted $534,312.50. What was the market quotation?

35. What income will a man receive from an investment of $6448 in United States coupon 4's at the lowest market price as per above table?

XX

INSURANCE

212. Nature of Insurance. Some losses or damages to property, such as by fire or tornado, are unavoidable. Such losses are serious, and they are often likely to bring financial ruin to owners. To avoid such calamities, owners of property subject to losses of some particular kind have frequently banded themselves together to share losses among themselves. By agreeing to assist in making good the loss to whomsoever it might fall, they secured themselves against disaster. From such beginnings has grown the institution of modern insurance. *Insurance*, then, consists of a contract guaranteeing to make good a loss from a certain cause. The agreement is known as an insurance *policy*.

213. Kinds of Companies. When an agreement for insurance is made between those mutually interested, to mutually share losses from a particular cause, it is known as *mutual insurance*.

When, as a business investment, a company is organized which undertakes, for a given fee, to make good a loss during a specified time, it is a *stock insurance company*. Sometimes a stock company agrees to divide all earnings of the company, above a specified dividend on their stock, among the insured. It thus partakes of the nature of both stock and mutual companies.

214. Property and Personal Insurance. Whenever it is a loss in property which is insured against, it is *property insurance*. Whenever the insurance is against a loss due to sickness, accident to, or death of a person, it is *personal insurance*.

PROPERTY INSURANCE

215. Forms of Property Insurance. If property is insured against loss from fire, it is *fire insurance;* if against loss from wind or storm, it is *tornado insurance;* if against loss or damage while being transported by land or by sea, it is *transit insurance*, that for ship or cargo lost at sea being *marine insurance;* if against loss or damage to live stock by death, disease, lightning, or other casualty, it is *live-stock insurance.* These are among the principal forms of property insurance.

216. Kinds of Policies. If the value of the property insured or the amount of the indemnity in case of loss is determined and agreed upon, at the time the policy is issued, it is known as a *closed* or *valued policy.* If the real value of the property loss is open for determination after the loss occurs, regardless of the face of the policy upon which the insurance premium has been paid, it is an *open policy.* Many states, by law, have declared that all policies must be valued policies. This requires the insurance company to pay the full face of the policy in case of a total loss, regardless of the actual value of the property.

217. Cost of Insurance. In mutual companies, the cost of insurance depends upon the amount of losses suffered by the different members of the company. This, together with the actual cost of carrying on the business of the company, is apportioned among the members, in the form of *assessments.* In stock companies, a definite fee is charged for insurance during a given period of time, and this fee is called a *premium.* The amount of the premium is usually a per cent of the face of the policy, or the amount of loss which it is agreed to make good. This is known as the *rate of insurance.* It varies with the kind of buildings, their location with reference to other buildings, efficiency of fire protection, etc.

Usually, a given district is plotted, and the rate for each building is fixed.

Below is a schedule of rates for such a plotted district, expressed as a certain sum for each $100 of insurance. The diagram on which this is based is on page 234.

TABLE OF RATES

District No. _____ City of _____

RISK	PLOT NO.	ANNUAL RATE PER $100
Frame Dwelling and Contents	1	$ .35
Brick Store and Contents	2	.25
Brick Church and Contents	3	.50
Brick Business Block and Contents	4	.50
Frame Handle Factory and Contents	5	1.75
Brick Livery Barn	6	1.25
Frame Store and Dwelling, and Contents	7	.40
Frame Barn and Contents	8	1.00
Brick Dwelling and Contents	9	.18
Brick Schoolhouse and Contents	10	.50
Brick Store and Dwelling, and Contents	11	.30

PROBLEMS

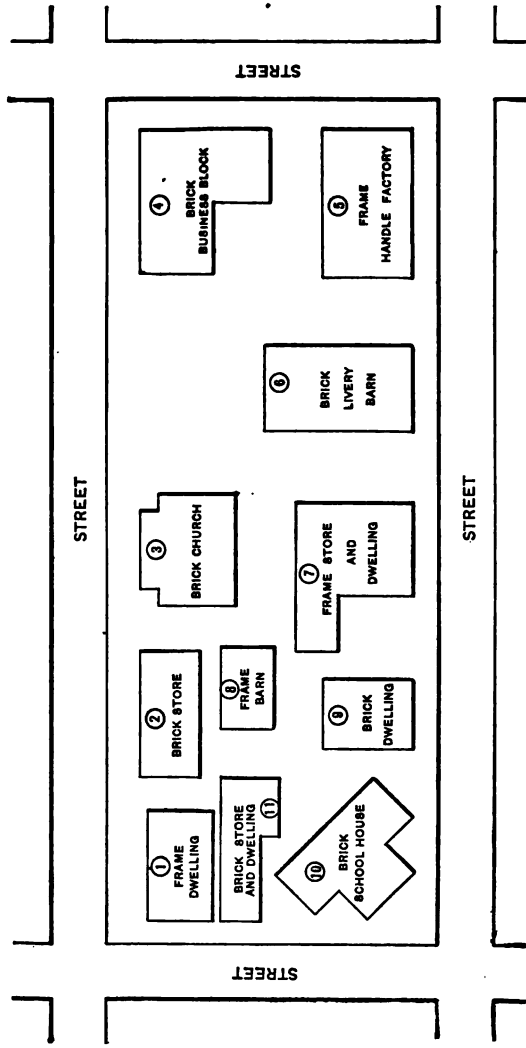
1. Find the cost of insuring property valued at $5000, at 1½%.

2. At 2%, how much insurance can I procure for $148?

3. I paid $30 for insuring a house worth $3200 at ¾ its valuation. What was the rate?

4. Find the cost of insuring each of the buildings in the table above at ¾ the following valuation:

Frame Dwelling	$2,400	Frame Store & Dwelling	$2,450
Brick Store	10,500	Frame Barn on Above Lot	500
Brick Church	12,000	Brick Dwelling	6,700
Brick Business Block	32,000	Frame Barn on Above Lot	800
Frame Handle Factory	4,500	Brick Schoolhouse	75,000
Brick Livery Barn	3,600	Brick Store & Dwelling	7,500



A PLOTTED DISTRICT.

5. The contents of the frame dwelling of the plot are valued at $1100; the contents of the brick store, at $12,450; the contents of the brick church, at $5500; the contents of the brick business block, at $25,600. If these goods are insured for $\frac{4}{5}$ of their valuation, what would be the premium on each lot?

6. An agent's premium is $25 for insuring a house at $\frac{1}{2}\%$. What is the face of the policy?

7. At $\frac{1}{2}\%$, what is the annual premium, at $\frac{4}{5}$ valuation, on a house worth $16,000?

8. A merchant pays $150 for insurance on his stock of goods at $\frac{3}{4}\%$. What is the amount of the policy?

9. An agent receives $112.50 for insuring a house for 80% of its valuation at $\frac{3}{4}\%$. What is the value of the house?

10. A man has a house valued at $24,000, and furniture valued at $6000. He insures the house at $\frac{4}{5}$ its valuation, and the furniture at $\frac{2}{3}$ its valuation. What is the annual premium, $\frac{3}{4}\%$ for the house and $\frac{1}{2}\%$ for the furniture?

11. If it cost $663 to insure a building valued at $132,600, what will it cost at the same rate to insure a building valued at $105,000?

12. The premium on a house valued at $10,500, insured at $\frac{5}{6}$ its valuation, is $47.25. Find the rate of insurance.

13. If a house and its contents are valued at $6500, for how much must it be insured, at $1\frac{1}{4}\%$, to cover loss and premium in case of total destruction?

14. A cargo of coffee valued at $35,000 was insured for $20,000, in a policy containing an average clause. It was damaged to the amount of $15,000. How much should the company pay?

NOTE. — Under an "average clause" such a part of the loss is paid, as the policy is of the real value of the property insured.

15. A steamer is insured for $75,000. Its value is $100,000. If it is insured at $2\frac{1}{2}\%$, what will be the loss to the company in case of total destruction?

16. An agent insures a cargo of cotton costing $9775, at the rate of $2\frac{1}{4}\%$, for an amount that will cover the cost of cotton and premium. What is the face of the policy?

17. A speculator bought 4000 bbl. of flour, and had it insured for 80% of its cost, at $3\frac{1}{2}\%$. He paid a premium of $980. At what price per barrel must he sell it in order to gain 10% on the total cost?

18. A manufacturer insured his factory for $27,000, and its contents for $66,000. He paid $700 for premium and policy. If the policy cost $2.50, what was the rate per cent premium?

19. A dealer in New York ordered his Chicago agent to buy 4000 bu. of wheat at 70¢; 3000 bu. at 30¢; 7500 bu. corn at 37½¢; paying 2½% commission for buying. The grain was shipped by boat, and a policy at 1½% was taken out to cover the cost of grain and commission. What was the amount of the policy, and what the amount of premium?

20. The Atlas Insurance Company insured a block of buildings for $150,000, at 75¢ per $100. Thinking the risk too great, it reinsured $50,000 in the Ætna, at ¾% and $65,000 in the Manhattan, at ¾%. How much premium did each company receive? What was the gain or loss to the Atlas? What per cent premium did it receive for the part of the risk not reinsured?

21. A house cost $8000; it was insured for ¾ its valuation at 1½% for 3 years. What would be my loss and that of the company, if the house were totally destroyed by fire?

22. A residence valued at $4500 is insured for ⅔ its value at ¾% per annum. The company will insure the house for 3 years on a payment of 2½ times the annual premium in advance. What will it cost to insure the house for 3 years? What will it cost to insure for 5 years, if the company will accept 4 annual premiums in advance as payment for 5 years?

23. How much will it cost to insure a factory valued at $75,000 at ¼% and the machinery valued at $25,000 at ⅓%?

PERSONAL INSURANCE

218. Life Insurance. When insurance is upon the hazard of life, it is *life insurance*. Upon the death of the insured it is paid to an heir or other person named in the policy as the *beneficiary*.

Life insurance may be paid for either by *assessments* when deaths occur, by a stated number of assessments in a year, or by a fixed *premium*, payable monthly, quarterly, semi-annually, or annually. When a fixed premium is paid, the policy may be either *participating* or *nonparticipating* in the profits of the association. The premiums may be payable annually

until death of the insured, a *life payment policy;* or may be payable annually only for a period of 10, 15, or 20 years, a *limited payment policy*, when the policy is said to be "fully paid up." The latter are termed 10 payment life, 20 payment life, etc.

What is known as a *term policy* may also be purchased for a period of 1, 5, 10, 15 or 20 years. The holder is insured for the term only, and, if desirous of continuing the insurance, must purchase another policy at increased rates, because of increased age.

When a fixed premium is charged, the insurance is said to be "old line." If the policy is a participating one, the profits of the company may operate to lessen the annual premium, or may be deferred until the policy matures and added to the face value.

Fraternal organizations, with a side feature of insurance or with insurance as their chief purpose, offer one method of life insurance, often with additional features of health or accident benefits. They usually employ the assessment plan, and often assess for more than enough to pay the death losses, in order to use the surplus in establishing a *reserve fund* to assist in paying death losses when the membership becomes older, and the percentage of deaths increase.

219. Endowment Insurance. *Endowment insurance* is a modern outgrowth of the life insurance idea. At the end of the stated endowment term, *e.g.* 5, 10, 15, or 20 years, the insurance is to be paid to the insured himself, should he be alive at that time. Should he die before that time, the full face of the policy is to be paid to the beneficiary named in the policy. While retaining the life insurance idea, it thus adds to it a savings or investment feature.

220. Accident and Health Insurance. *Accident* or *casualty insurance* companies pay an indemnity when one is injured

by an accident, in travel or otherwise. The amount paid is usually graduated to the extent of the injury.

Insurance against ill health may also be had, the insured receiving a weekly or monthly payment while sick.

221. Cost of Life Insurance. The price of a life insurance policy is a stated annual amount or premium per thousand

ANNUAL PREMIUM RATES FOR $1000 INSURANCE

WHOLE LIFE					ENDOWMENT PREMIUM		
AGE	LIFE	20 YEARS	15 YEARS	10 YEARS	AGE	IN 15 YEARS	IN 20 YEARS
21	$19.47	$29.59	$35.65	$48.11	21	$67.03	$49.07
22	19.91	30.06	36.20	48.85	22	67.13	49.17
23	20.36	30.55	36.78	49.61	23	67.23	49.27
24	20.84	31.06	37.38	50.40	24	67.33	49.39
25	21.34	31.58	38.00	51.22	25	67.44	49.52
26	21.86	32.12	38.63	52.06	26	67.56	49.65
27	22.41	32.69	39.63	52.93	27	67.69	49.79
28	22.99	33.27	39.98	53.83	28	67 83	49.95
29	23.59	33.88	40.70	54.76	29	67.97	50.11
30	24.23	34.51	41.43	55.73	30	68.12	50.28
31	24.87	35.14	42.17	56.70	31	68.29	50.48
32	25.55	35.81	42.94	57.71	32	68.46	50.69
33	26.30	36.52	43.76	58.78	33	68.66	50.91
34	27.08	37.26	44.62	59.88	34	68.87	51.16
35	27.91	38.04	45.51	61.03	35	69.09	51.42
36	28.78	38.85	46.43	62.21	36	69.34	51.72
37	29.70	39.70	47.39	63.44	37	69.60	52.04
38	30.68	40.59	48.39	64.71	38	69.90	52.40
39	31.71	41.51	49.43	66.03	39	70.21	52.79
40	32.81	42.49	50.52	67.40	40	70.56	53.22
41	33.94	43.50	51.63	68.80	41	70.95	53.70
42	35.14	44.55	52.79	70.24	42	71.37	54.22
43	36.45	45.69	54.04	71.77	43	71.85	54.81
44	37.83	46.90	55.34	73.36	44	72.37	55.46
45	39.30	48.17	56.71	75.02	45	72.95	56.17

dollars of insurance. The lowest in cost is in purely mutual companies, like fraternal organizations.

The price depends, too, upon the age of the applicant and upon the conditions of the contract. The term policy, or the straight life-payment nonparticipating policy may be purchased at the least cost, while a limited payment or endowment policy will cost proportionately more. Rates vary, but the table on the opposite page gives a fair approximation of the cost of the common kinds of policies in standard companies at different ages.

PROBLEMS

From the table find the annual premium required for:

1. A life policy of $3000, age 30.

2. A twenty-payment life policy of $5000, age 27.

3. A twenty-year endowment policy for $4000, age 32.

4. A ten-payment life policy for $3500, age 35.

5. A man takes out a twenty-payment life policy for $3000 at the age of 25. If he dies at the age of 40, how much does the face of the policy exceed the premiums paid?

6. If money is worth 6%, what do the premiums in problem 5 amount to? How much does the face exceed that amount? (Annual interest.)

7. A man at the age of 28 takes out a straight life policy and a twenty-year endowment policy, each for $2000. If he dies at 40, which gives the greater returns?

8. A man 30 years of age took out an endowment policy for $3000, payable in 15 years, and died after making six payments. How much less would a life policy have cost?

9. A man aged 35 years takes out an endowment policy for $15,000, payable to himself in 20 years, or to his heirs if he dies before that time. What annual premium will he have to pay? If death occurs at the end of the ninth year, how much would he have paid out in premiums? How much less would a twenty-payment life policy have cost?

10. At the age of 32 a man takes out a $3500 life policy, and at the age of 35 a $1000 twenty-year endowment. How much does the insurance exceed the premiums paid, if he dies at the age of 45?

XXI

PROPORTION

222. Ratio. The relative size of two numbers, as shown by division and expressed by their quotient, is called their *ratio*. Thus the ratio of 8 to 4 is 2; 96 to 60 is $1\frac{3}{5}$; and of 25 to 50 is $\frac{1}{2}$. Instead of using the sign of division ($\div$) to express a ratio, the horizontal line is left out, making the sign simply a colon (:), *e.g.* 8 : 2 or 96 : 60. The meaning, however, remains the same, and the ratio may always be found by dividing the first number or *term* by the second.

223. Comparison of Like Quantities Only. Since ratio shows comparative size, it may only exist between quantities like in kind. Thus, 8 ft. and 4 ft., or 8 houses and 4 houses, may be compared, while 8 boxes and 4 horses may not. 8 ft. and 4 yd. may be compared, but, to do so, they must first be reduced either to feet or yards, *e.g.* the ratio of 8 ft. to 12 ft., or $\frac{2}{3}$.

When the numbers refer to units of measurement, the quantities being measured must also be like in kind, if the ratio of the quantities being measured is desired. Thus, 8 doz. and 2 doz. may be compared, if it is 8 doz. chairs and 2 doz. chairs, but not if 8 doz. chairs and 2 doz. horses; or 8 ft. and 2 ft. may have ratio if it is 8 ft. high and 2 ft. high, but not if 8 ft. high and 2 ft. wide, etc.

224. Order of Terms. The number to be treated as a dividend is always written first. The ratio of 6 : 3, then, would be 2 and *not* $\frac{1}{2}$. Since the number of which it is

desired to know the comparative size is always written first, it is called the *antecedent*, *i.e.* the one which goes before. The number with which it is desired to compare the first is always written second, and because it follows the other, it is called the *consequent*. The antecedent and its consequent together form a *couplet*.

225. Proportion. The ratio of two different couplets may be the same. Thus, the ratio of 8 horses to 2 horses is 4, and the ratio of $400 to $100 is 4. Likewise, the ratio of 3 ft. in height to 6 ft. in height is $\frac{1}{2}$, and the ratio of 3 days to 6 days is $\frac{1}{2}$.

Whenever two ratios are equal, they are in *proportion*. Thus, as the ratio of 8 horses to 2 horses is 4, the ratio of their cost ($400 to $100) would also be 4. We would, therefore, say that the ratio of 8 horses to 2 horses equals the ratio of $400 to $100.

The equality of related ratios is usually expressed by the double colon, *e.g.* 8 horses : 2 horses :: $400 : $100. This proportion would be read as follows: 8 horses is to 2 horses as $400 is to $100. The equality sign may also be used, giving the above proportion the form of 8 horses : 2 horses = $400 : $100.

226. Means and Extremes. As there must be two ratios whose equality forms a proportion, every proportion must have four *terms*. When written formally as a proportion, the first and fourth terms are called the *extremes*, meaning the outside numbers. The second and third terms of a proportion are called the *means*, meaning the middle numbers.

Since the antecedents of each ratio are the dividends, and the consequents are divisors, the extremes and means each consist of one dividend and one divisor. The quotient of each couplet being the same, the product of the dividend of one couplet and the divisor of the other is equal to the prod-

uct of the other dividend and divisor. In other words, *the product of the means is equal to the product of the extremes.*

The above being true, it is only necessary to know any three of the terms of a proportion in order to find the fourth. Thus, in the proportion, 2 hats : 5 hats :: \$3 : \$$x$, the product of the means (treating the terms of the first ratio as multipliers and, therefore, abstract numbers) is \$15. Since this \$15 is the product also of the extremes, and one of them is 2, the other must be $\frac{1}{2}$ of \$15, or \$7.50. The completed proportion, then, would be 2 hats : 5 hats :: \$3 : \$7.50.

The use of three terms to find the fourth, has given rise to the phrase "the rule of three," which is a name formerly applied to solutions by proportion.

227. Directly and Inversely Proportional. Assuming that each man does an average day's work, the larger the number of men employed on a given piece of work, the more work is done. Thus, if 2 men can wrap and pack 150 boxes of oranges in a day, 4 men can wrap and pack 300 boxes. In other words, if the number of men is doubled, twice the work is accomplished. Whenever two quantities increase or decrease together in this way, and with a constant ratio, they are said to *vary directly* or to be *directly proportional.* We may also say that the amount of work done would bear a *direct ratio* to the number of men employed.

On the other hand, if the number of men is increased, the same amount of work would require less time for its completion. Thus, if 2 men can wrap and pack 600 boxes in 4 days, 4 men could do it in half the time. In other words, if the number of men is doubled, the time required is but one half as much. Whenever one quantity increases as another decreases, or decreases as another increases, keeping the ratio constant, they are said to *vary inversely*, or to be *inversely proportional.* Thus, the time required to do a given piece

of work bears an *inverse ratio* to the number of men employed.

QUERY. — Which of the following are directly and which inversely proportional:

1. The weight of coal and its cost?
2. The height of buildings and their shadows?
3. The number of workmen and the amount of work done in a given time?
4. The number of workmen and the time required to do a given amount of work?
5. The speed of an automobile and the time required to travel a certain distance?
6. The weight of freight and the freight charges?
7. The area of squares and the length of their sides?
8. The amounts loaned and interest earned?
9. The attraction one body has for another and their distance apart?
10. The greater the capital stock in a company and the size of the dividend which can be declared with given profits?
11. The amount of assessable property and the tax levies to raise a given amount?
12. The amounts loaned and the rates to earn the same amount of interest?

228. Statement and Solution. The direct proportion given in Section 227 would be stated thus:

4 men : 2 men :: 300 boxes : 150 boxes.

In the second couplet, 300 boxes, or what is done by 4 men, is the first term, just as 4 men is the first term of the first couplet; and 150 boxes is, likewise, the second term of the second couplet, as 2 men is the second term in its couplet.

The inverse proportion would be stated thus:

4 men : 2 men :: 4 days : 2 days.

This order is necessary that the ratio of each couplet be the same (in this case, 2). It will be noticed, however, in the second couplet, that 2 days, the time required by 4 men, is the *second* term in its couplet, although the 4 men is the *first* term in its couplet. Likewise, the 4 days is the *first* term of its couplet, although the two men working is the *second* term. In other words, in direct proportion the *order of the terms in the second couplet is the same as in the first.*

In inverse proportion the *order of the terms in the second couplet is the inverse of the order in the first.*

NOTE. — As a matter of convenience and simplicity in solution, it will be found best to use the couplet which contains the unknown or required term as the first couplet and the unknown term as its second term.

1. 2 men : ? men : : 150 boxes : 300 boxes.

300 boxes × 2 = 600 boxes.

600 boxes ÷ 150 boxes = 4, or the number of men required.

2. 2 men : ? men : : 2 days : 4 days.

4 days × 2 = 8 days.

8 days ÷ 2 days = 4, the number of men required.

In general, to solve a proportion when it is correctly stated:

1. *Divide the product of the two given means by the one given extreme*, or

2. *Divide the product of the two given extremes by the one given mean.*

229. Compound Proportion. Whenever two or more ratios are equal to another ratio, the proportion is said to be *compound.* Compound proportion, therefore, involves three or more couplets, all having the same ratio.

The statement of problems containing a compound proportion requires the comparing of every couplet to one basal couplet (usually taken as the first couplet), ascertaining whether the proportion be direct or inverse, and arranging the terms of the second couplets accordingly.

230. Arranging in Couplets. As an aid to comparing the couplets and to stating the proportions involved, it will be found helpful to arrange first all terms given in the problem, in their proper couplets.

PROBLEM. — If 90 men have completed the construction of 3 miles of railroad in 80 days, how many men should be engaged to fulfill a contract to build 10 miles of road in 150 days?

1. Write all the terms belonging together in a column, thus,

90 men
3 miles
80 days

2. On the other side of a vertical line drawn to the right of the column, write the remaining terms, arranging like opposite like, forming couplets, thus,

90 men	?
3 miles	10
80 days	150

231. Statement of a Compound Proportion. Using the couplet containing the unknown term as the first couplet, the following couplets should be compared to it and the terms of each written as a part of the second or compound couplet, their order depending on whether the statement is of direct or inverse proportion. Thus, in the example given, if the number of men were increased, the number of miles that could be built would also increase. It is, therefore, a

direct proportion, and the order in both couplets would be the same, *e.g.*

90 men : ? men : : 3 miles : 10 miles.

But if the number of men were increased, it would take a *less* number of days to do the work. This proportion would, therefore, be inverse, and the second couplet would be written in inverse order, thus,

90 men : ? men : : 150 days : 80 days.

Combining these two proportions into a compound proportion, we have:

STATEMENT

$$90 \text{ men} : ? \text{ men} : : \begin{matrix} 3 \text{ miles} : 10 \text{ miles} \\ 150 \text{ days} : 80 \text{ days.} \end{matrix}$$

232. Solving Compound Proportion.

10 miles × 80 × 90 = 72,000 miles.
3 miles × 150 = 450 miles.
72,000 miles ÷ 450 miles = 160, or the number of men required.

SOLUTION BY CANCELLATION

$$\begin{matrix} 2 \\ \cancel{6} \\ \cancel{90} \end{matrix} \text{ men} : ? \text{ men} : : \begin{matrix} \cancel{3} \text{ miles} : \cancel{10} \text{ miles} \\ \cancel{150} \text{ days} : 80 \text{ days.} \\ \cancel{15} \end{matrix}$$

2 men × 80 = 160 men. *Ans.*

NOTE. — All factors removed by cancellation must be taken out of *both* the means and the extremes.

PROBLEMS

1. What effect does the multiplying or dividing of both terms of a ratio have on its value?

2. What effect does multiplying or dividing the antecedent have on the ratio? Multiplying or dividing the consequent?

What is the ratio of:

3. \$8 to \$2? \$¼ to \$½? \$.50 to \$.12½? \$26 to \$5.20?

4. 100 to 25? 100 to 14⅔? 33⅓ to 100? 100 to 6⅔?

5. \$.50 to \$.15? 2 m. to 40 cm.? 15 hr. to a day?

6. What number has to 40 the ratio of 2? Of ½? To 5 the ratio of 5? To 15 the ratio of ⅓? Of 3? To 84 of 7? Of ⅐?

7. 28 has the ratio of 2 to what number? 12½ has the ratio of ⅛ to what number? ⅜ has the ratio of 3 to what number?

8. A pound of coffee costs 40¢, and of butter 25¢. What was the ratio of their costs?

9. The diameter of a circle is 7 ft. and the circumference is 22 ft. What is the ratio of the circumference to the diameter?

10. If a map is drawn on a scale of 1 in. to 1 mi., in what ratio are the dimensions diminished?

11. One door is 6 ft. 6 in. by 3 ft. 8 in.; another is 7 ft. 6 in. by 4 ft. What is the ratio of the first to the second? Of the second to the first?

Examine each of the problems from 12 to 30, and tell whether the ratio is direct or inverse. Solve.

12. If 6 horses cost \$1200, what will 10 horses cost at the same rate?

13. If 12 yd. of cloth cost \$20, what will 35 yd. cost?

14. If 6 horses cost \$300, how many can be bought for \$900?

15. If 15 yd. of silk cost \$22.50, how many yards can be bought for \$36?

16. If it takes 48 yd. of carpet 1 yd. wide to cover a floor, how many yards will it take of carpet ¾ yd. wide?

17. \$200 earns \$12 interest. How much interest will \$350 earn?

18. A merchant pays \$6 freight on 1000 lb. of merchandise. What rate is that per 100 lb.?

19. When coal is worth \$9 a ton, what will 1200 lb. cost?

20. A man with an income of \$1000 saved \$300. The next year his income was \$1200 and he saved a proportional amount. How much did he save?

21. At a certain time of day, a post 4 ft. high casts a shadow 3 ft. long. What is the height of a tree that casts a shadow of 15 ft.?

22. A pipe discharging 6 gal. a minute can fill a cistern in 4 hr. How long will it take a pipe discharging 8 gal. a minute to empty it?

23. If a man sells $\frac{3}{4}$ of his farm for $4200, what would $\frac{1}{4}$ of it be worth at the same rate?

24. A hall was paved with tiles 9 in. square, and 640 were used. How many tiles 6 in. square would it take?

25. If a tower 40 ft. high casts a shadow 70 ft. long, how long a shadow will a tower 110 ft. high cast?

26. If it cost $60 to make a walk 10 ft. wide and 180 ft. long, how much will it cost to make a walk 8 ft. wide and 450 ft. long?

27. If 3000 bricks, each 8 in. long and 4 in. wide, will lay a walk, how many bricks 6 in. square would it take?

28. If it cost $168 to roof a space 72 ft. long and 21 ft. wide, how much will it cost to roof a space 66 ft. long and 27 ft. wide?

29. If 170 bu. of oats feed 120 horses 34 days, how long would 150 bu. feed 90 horses?

30. If 12 men, in 4 da. of 8 hr. each, earn $152.60, at the same rate, how much will 22 men earn in 5 da. of 9 hr. each?

31. 10 men can pave a street 30 ft. long and 48 ft. wide in 2 da. How many men will it take to pave a street 400 ft. long and 36 ft. wide in $12\frac{1}{2}$ da.?

32. If $290.70 interest accrues on $1020 at 6% for 4 yr. 9 mo., how much interest must be paid on $2700 at $7\frac{1}{2}$% for 3 yr. 4 mo.?

33. If $675, put at interest at 8‰ earns $9 interest in 60 da., in how many days will $1240 earn $28.80 interest at 6%?

34. If 20 men working 12 da. of 8 hr. each can cut 400 cd. of wood, how many cords should 12 men cut in 15 da. of 10 hr. each?

35. If a piece of timber 11 ft. long, 10 in. wide, and 8 in. thick weighs 1848 lb., find the length of another piece of timber which weighs 6048 lb., and which is 6 by 24 in.?

36. If 10 horses eat 16 bu. 16 qt. oats in 9 da., how many days, at the same rate, will 123 bu. 28 qt. feed 34 horses?

37. If 21 men can build a wall 28 rd. long in 96 da., how many men will be required to build $31\frac{1}{2}$ rd. in 84 da.?

38. If 12 compositors in 60 da. of 10 hr. each set up 50 sheets of 16 pages each, 32 lines on a page, in how many days of 8 hr. can 18 compositors set up, in the same type, 128 sheets of 12 pages each, 40 lines to the page?

39. A contractor engaged to lay 20 mi. of road in 300 da. At the end of 80 da. he finds that 90 men have laid 3 mi. How many more men must he engage to finish the work in the required time?

40. If 54 T. of anthracite coal can be stored in a bin $28 \times 20 \times 4$ ft., how many tons can be stored in a bin $45 \times 18 \times 8$ ft.?

41. If a mow $10 \times 6 \times 8$ yd. holds 32 T. of hay, how deep must a mow be that is 24 ft. long and 15 ft. wide, in order to hold 86 T.?

42. If 60 men make an embankment $\frac{1}{4}$ of a mile long, 30 yd. wide, and 7 yd. high in 42 da., how many men will it take to make an embankment 1000 by 36 yd. and 22 ft. high in 30 da.?

43. If 50 men can do a piece of work in 48 da. working 8 hr. a day, how many hours a day would 50 men have to work in order to do the same work in 32 da.?

44. If the interest on \$84 at 6% for 3 yr. is \$15.12, what sum must be loaned at 8% for 1 yr. 6 mo., to earn the same amount?

45. If I loan \$600 for 8 mo. and get \$20 interest, for what time must I loan \$1200 at the same rate to get \$90 interest?

XXII

PROPORTIONAL PARTS AND PARTNERSHIP

233. Partnership. The association of two or more persons in a business firm, or *partnership*, has already been outlined under Stocks and Bonds (Sec. 196). During the progress of a business, it often becomes necessary to take an inventory of the financial condition of the firm. For this purpose, a statement of resources and liabilities is made. Under *resources* are listed all property on hand and all accounts owed to the business, and under *liabilities* all debts of the firm. This casting up of accounts will show whether the profits of the business exceed the expenses (*net gain*) or whether the expenses have been greater than the profits (*net loss*). It will also show whether the firm has sufficient resources to meet all liabilities, in which case it is *solvent*, or if its liabilities are greater than its resources, when it is *insolvent*. From these statements, too, may be computed the present financial condition of the firm (the net resources after allowing for all liabilities), which is termed the *present worth*.

234. An Application of Proportion. The distribution of the profits or apportionment of the losses of a business partnership is often so simple as to be easily resolved into fractional parts. The conditions of partnership may, however, become quite complicated, involving different forms and amounts of investments, withdrawals, and increases, different lengths of time, etc. In such cases the apportionment of profits or losses are often easily and quickly accomplished by proportion.

235. Partitive Proportion. The form of proportion used for that purpose is known as *partitive proportion* or *proportional parts*. The term "partitive proportion" means partitioning a whole into parts, proportionally. The parts of the profits belonging to each partner would bear the same ratio, *i.e.* be proportional to the investments made, the time the capital was used, or some other definite ratio that may be ascertained.

Thus, if A invested \$500, and B invested \$1000, a profit of \$750 would be divided into \$250 and \$500, respectively. The sum of all the parts invested would be \$1500, and A would be entitled to $\frac{500}{1500}$ (or $\frac{1}{3}$) of all the profits, which would be \$250.

Stated proportionally, A's profit would be found by solving the proportion

$$\$500 : \$1500 :: ? : \$750$$
$$\$375{,}000 \div \$1500 = 250, \text{ or } \$250 \text{ profit.}$$

B's profit would be:

$$\$1000 : \$1500 :: ? : \$750$$
$$\frac{\$750{,}000}{\$1500} = 500, \text{ or } \$500 \text{ profit.}$$

236. Equivalent Investments. When investments in the business are made for different lengths of time, the profits or losses are often distributed in proportion to the equivalent investments. By *equivalent investment* is meant the sum which, invested for a unit of time, is equivalent to various sums invested for different periods of time.

Thus, if A's \$500 were invested for 6 mo. and B's \$1000 for 4 mo., the profit of \$750 would not be distributed as $\frac{1}{2}$ and $\frac{1}{2}$. A's \$500, invested for 6 mo., would be equivalent to \$3000, invested for 1 mo.; and B's to \$4000, invested for 1 mo. Their total equivalent investments, then, would be \$7000, and A's profits would be $\frac{3}{7}$ of \$750, or \$321.43.

Proportionally solved, A's profits would be

$3000 : $7000 :: ? : $750

$2,250,000 ÷ $7000 = 321.43. or $321.43 profit.

When a partner's capital is increased or decreased during the term he remains a partner, his equivalent investment is found by finding equivalent investments for a unit of time for each different capital, and adding such equivalent investments.

Thus, if $1000 was invested, and increased $500 after 4 mo., and again increased $500 after another 2 mo., and decreased $800, 6 mo. later, where it remained for 10 mo. longer before there was a distribution of profits, equivalent investments would be found in the following way:

$1000 × 4 = $4,000, equivalent investment for 1 mo.
1500 × 2 = 3,000, equivalent investment for 1 mo.
2000 × 6 = 12,000, equivalent investment for 1 mo.
1200 × 10 = 12,000, equivalent investment for 1 mo.
$31,000 equivalent investment for 1 mo.

237. Adjustments by Interest. Inequalities in amounts and time of investments, especially when there are increases and withdrawals of investments at irregular periods, are often adjusted by allowing interest on all investments and charging interest on all sums withdrawn. The profits or losses remaining after interest has been allowed or charged, may then be divided equally or according to any fixed ratio.

Thus at 6% A's $500 would bear $15 interest in 6 mo. If $200 were withdrawn at that time for the remaining 6 mo., his $300 would earn $9 in the remaining 6 mo., and he would be charged $6 interest on such withdrawal. His net interest earned would, therefore, be $15 + $9 − $6 = $18. B's $1000 for 4 mo. would be allowed $20 interest, and $400 for 8 mo., $16, and if $600 were withdrawn, he would

be charged $24 for the remaining 8 mo., leaving a net interest earning of $12. A's earned interest of $18 and B's of $12 must first be paid out of the $750 profit, leaving $720 to be divided in proportion to the original investment. Of this, A would get $\frac{1}{3}$, or $240, plus his interest of $18, or $258, and B would get $\frac{2}{3}$, or $480, plus his interest of $12, or $492.

NOTE. — When interest is allowed and charged on capital increases and withdrawals, net profits are often shared equally, after interest has been paid.

PROBLEMS

1. Divide 360 into parts proportional to 3 and 6.

2. Divide $800 into parts proportional to 1, 3, and 6.

3. Divide $240 into parts proportional to $\frac{1}{2}$ and $\frac{1}{3}$.

4. Divide $780 among three persons, whose shares will be in proportion to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

5. A, B, and C engage in business for 1 yr. A puts in $5000, B $3000, and C $2000. If they gain $3600, what is each man's share?

6. Divide $2400 among A, B, and C, so that A's part will be twice C's and $\frac{1}{2}$ B's.

7. The total receipts of a gold mining company for 1 yr. were $15,750,000. The expenses were to the net earnings as 12 to 3. What were the expenses? The net earnings?

8. Divide the simple interest on $65,000 for 1 yr. 8 mo., at 5$\frac{1}{2}$%, among A, B, and C, so that A's part is 8 times C's and $\frac{1}{3}$ B's.

9. A, B, and C pay $75.60 for a pasture. A puts in 10 horses, B 24 cows, and C 120 sheep. If 3 sheep eat as much as 1 cow and 2 cows as much as 3 horses, what rent must each pay?

10. The annual earnings of a steamship company were $39,000,000. Find the amounts received from freight charges, from passenger service, and from other sources, if they were in the proportion of 7 : 4 : 2.

11. The holdings of the shareholders of a corporation are 32, 13, 22, 20, 50, 34, 42, and 72 shares, respectively. If a dividend of $12,825 is divided among them, what does each shareholder receive?

12. C and D engaged in business and gained $3500. C's capital was $8000, and D's was $6000. Find each partner's share, if the profits were divided according to investment.

13. Two men owned a carriage factory. One had invested $75,000, the other $45,000. The net earnings for 1 yr. were $12,200. What was each partner's share?

14. A, B, and C enter into partnership with a joint capital of $130,000; A furnishes $\frac{1}{2}$, B $\frac{1}{3}$, and C the remainder. Their net gain is 30% of the amount invested. Find each man's share of the gain.

15. Dunn and McDonald formed a partnership in which Dunn invested $5000 and McDonald invested $2500. The gains were: merchandise, $940.25; real estate, $356.50; losses, expense, $420. What was the net gain? What was the gain of each partner? What was each partner's present worth at the close of the business?

16. A and B formed a partnership with a capital of $10,000. A furnishes $4000 and B $6000. After 16 mo. A withdrew $500, and at the end of 18 mo. B withdrew $1000. At the end of 2 yr. the partnership was dissolved and a profit of $8750 was divided. How much did each partner receive?

17. A and B engaged in a dry goods business for 3 yr. from April 1, 1907. Each invested $1800. July 1, 1907, A increased his investment $350, and B withdrew $300; Feb. 1, 1908, each withdrew $800; Feb. 1, 1909, each invested $1200. There was a gain of $1800 on March 1. How should it be divided?

18. C and D formed a partnership, C investing $9000, and B $12,000. It was agreed that B should take $3000 from the gains before a division was made, and that the net gain or loss should then be shared equally. The gains were $7580 and the losses $1275. What was the net gain of each partner? The present worth at dissolution?

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